<u>Stage 2 Modules in this section</u>

MAS2101	Modelling with Differential Equations
MAS2103	Complex Variable
MAS2104	Introduction to Vector calculus
MAS2105	Methods for Solving Differential Equations
MAS2106	Chaos: Theoretical & Numerical Methods
MAS2201	Applications of Mathematical Methods to Finance
MAS2202	Number Systems and the Foundations of Analysis
MAS2211	Set and Functions
MAS2212	Linear Algebra and Analysis
MAS2213	Algebra
MAS2216	Enumeration and Combinatorics
MAS2301	Introduction to Probability and Statistics
MAS2302	Introduction to Statistical Inference
MAS2303	Applied Probability
MAS2304	Foundations of Probability
MAS2305	Statistical Inference

Modelling with Differential Equations MAS2101

Module Leader: **Prof A Shukurov**

Additional Lecturer(s): **Prof RS Johnson**

Credits: 20

Semester: 1 and 2

Programme: 43 lectures, 9 problem classes/practicals, 4 tutorials and 2 feedback sessions

Pre-requisites MAS1001
Co-requisites: None

Description

Much modern applied mathematics uses differential equations to describe (or model) phenomena of the real world. A good mathematical model is a tremendously powerful tool in both understanding and predicting the behaviour of physical processes. The applications of modelling are remarkably widespread, covering the physical, biological and social sciences. Examples of applications will be presented, taken from, for example, biology (e.g. predicting how a disease spreads through a population) and from the physical world (e.g. motion under gravity).

This course introduces the fundamental ideas of formulating, solving and interpreting mathematical models based on difference equations and, above all, on ordinary differential equations (ODEs). Although much of the course is devoted to mathematical methods for solving ODEs, some emphasis is placed on understanding the meaning and validity of mathematical solutions. In addition, techniques for the numerical solution of ordinary differential equations are introduced. These ideas will be illustrated using the computer algebra package Maple (introduced in MAS1001); this package makes it easier to learn how to solve ODEs, and to produce graphs.

Aims and Objectives

To present the techniques of mathematical modelling of the real world by the use and solution of ODEs; to introduce methods for the numerical solution of ODEs using Maple.

Students will be able to formulate and interpret simple problems in terms of ODEs, and solve them, either by analytical or numerical methods; they will gain more experience of Maple.

Syllabus

Review of vectors, including dot and cross products. Formulation of mathematical models in terms of differential and difference equations. Methods of solution of first-order ODEs: separable, integrating factor, homogeneous. Second-order ODEs: constant coefficients, variation of parameters, particular integrals. Line integrals, Green's theorem. Numerical analysis (implemented with Maple): Euler's explicit method, stability. Applications to, for example: fluid motion, population growth, Newtonian mechanics (gravity, resistance).

Reading

M. Braun: *Differential Equations and Their Applications* (Springer, 1992). (R)
T.P. Dreyer: *Modelling with Ordinary Differential Equations* (CRC Press, 1994). (R)
G. Fulford, P. Forrester and A. Jones: *Modelling with Differential and Difference Equations* (C.U.P., 1997). (R)
K.M. Heal, M.L. Hanson & K.M. Rickard: *Maple V Learning Guide* (Springer, 1996). (R)
A. Skidmore & M. Hale*: A Guided Tour of Differential Equations (using Computer Technology)* (Prentice-Hall, 1998). (R)

Assessment

Assessment of course work (20%).
1.5 hour examination at the end of each semester (each 40%).

Complex Variable MAS2103

Module Leader: **Prof I G Moss**

Credits: 10

Semester: 2

Programme: 24 lectures + 12 problem classes/drop-ins/ practicals

Pre-requisites MAS1001, MAS1101, MAS1202

Co-requisites: None

Description

The idea of a complex number, $z = x + iy$, is very familiar, as is its use to find solutions of algebraic equations with complex roots. However, the important and exciting ideas follow when we consider functions of a complex variable: $f(z)$. These functions turn out to have surprising and dramatic properties that are quite unexpected, when compared with real functions of real variables (such as $y = f(x)$). Indeed, in many respects, the natural way to discuss functions – all our familiar functions such as exponential and trigonometric functions, for example – is in the complex plane. We shall introduce a discussion of elementary functions in the complex plane, and their differentiability, and then move to the even more amazing results that follow when we investigate integration.

Integration in the complex plane involves the notion of a line integral, but most calculations can be reduced to a simple algebraic exercise – something which is quite counter-intuitive, and a surprising relief to those who have been brought up on conventional integration! Indeed, these methods can be used to tackle – very simply – many integrals that are difficult if approached using standard methods.

The theory of complex variables, and particularly the methods of integration, play an important role in many branches of pure and applied mathematics; the techniques are essential, for example, in any study of elementary fluid mechanics.

Aims and Objectives

To introduce the main results in the theory of analytic functions, together with some properties of specific functions, and the associated methods of integration.

Students will be able to manipulate the elementary functions in the complex plane, and to perform some contour integrals, a few of which will relate to the evaluation of real integrals.

Syllabus

Differentiability: Cauchy-Riemann relations; analytic functions; principal value. Contour integration; Cauchy's integral theorem; Cauchy's integral formula. Poles, residues, Laurent series; Cauchy's residue theorem. Evaluation of real integrals. Fourier transforms: evaluation.

Reading

L.V. Ahlfors, *Complex Analysis*, 3rd edition (McGraw-Hill, 1979). (B)
J.W. Brown & R.V. Churchill, *Complex Variables and Applications*, 7th edition (McGraw-Hill, 2004). (B)
A. Jeffrey, *Complex Analysis and Applications*, 2nd edition (CRC, 2006). (B)
E. Kreyszig, *Advanced Engineering Mathematics*, 8th edition (Wiley, 1999). (R)
A.D. Osborne, *Complex Variables and their Applications* (Addison-Wesley, 1999). (B)
H.A. Priestley, *Introduction to Complex Analysis*, (Clarendon Press, 1990). (R)

Assessment

Assessment of course work (10%).
1.5 hour examination at end of Semester (90%).

Introduction to Vector Calculus MAS2104

Module Leader: **Dr G R Sarson**

Credits:	10	Programme:	24 lectures + 12 problem classes/drop-ins/practicals
Semester:	1		
		Pre-requisites	MAS1001 and MAS1101
		Co-requisites:	None

Description

The development of mathematical representations of physical and thought models, and their solutions, requires some basic mathematical tools. This course (with MAS2105) introduces the various ideas that are needed in order to describe and formulate problems in three dimensions. Thus MAS2104 provides the important links between the calculus and (3D) vectors: the vector calculus.

Aims and Objectives

To present the basic mathematical methods needed in the formulation of both physical and mathematical problems involving vector and scalar quantities in 3D space.

Students will be able to describe various properties of vector fields, be familiar with the language of the vector calculus and know, and be able to apply in simple situations, the integral theorems.

Syllabus

Vector Calculus: scalar and vector functions and fields; gradient, divergence and curl; spherical and cylindrical coordinates; curves, tangent vectors and review of line integrals; surfaces, normal vectors, surface and volume integrals; Green's, Gauss' and Stokes' Theorems.

Reading

M.L. Boas: *Mathematical Methods in the Physical Sciences*, 2nd Edition, (Wiley). (R)
E. Kreyszig: *Advanced Engineering Mathematics*, 8th Edition (Wiley, 1999). (R)
E. Kreyszig & E.J. Norminton: *Maple computer guide: a self-contained introduction for Erwin Kreyszig 'Advanced Engineering Mathematics'*, 8th ed. (Wiley, 2000). (R)
K.F. Riley, M.P. Hobson & S.J. Beuce: *Mathematical Methods for Physics and Engineering*, (CUP). (R)
M.R. Spiegel: *Vector Analysis* (Schaum Outline, McGraw-Hill, 1974). (R)
K.A. Stroud & D.J. Booth: *Advanced Engineering Mathematics* (Palgrave). (R)

Assessment

Assessment of course work (10%).
1.5 hour examination at end of Semester (90%).

Methods for Solving Differential Equations MAS2105

Module Leader: **Dr A Fletcher**

Credits: 10

Semester: 2

Programme: 24 lectures + 12 problem classes/drop-ins/practicals

Pre-requisites MAS1001 and MAS1101
Co-requisites: None

Description

Almost all of physical nature, we argue, can be represented (often very accurately indeed) by a mathematical model; this can range over the realms of fluid flow, cosmology, quantum mechanics and neural networks (to mention four at random). In order to lay the foundations for studies of this sort, we first need to gain some experience and knowledge in the techniques for solving both ordinary and partial differential equations, since mathematical descriptions normally reduce to differential equations. This course introduces the essential elements in the theory of ordinary and partial differential equations, and their methods of solution, which will then provide the basis for specific studies in other modules. The standard ordinary and partial differential equations that are used in applied mathematics, physics and engineering will be described, but with the emphasis on their solution rather than on extensive physical interpretation (although this aspect certainly will be mentioned). The computer package Maple will be used to explore the properties of some of these ideas and, in particular, to investigate (numerically) the summation of series.

Aims and Objectives

To present some of the standard techniques for solving both ordinary and partial differential equations.

Students will acquire the ability to solve certain standard ODEs and PDEs, become familiar with a number of different techniques of solution and gain some experience in interpreting solutions.

Syllabus

Ordinary differential equations: series solutions, including an introduction; introduction to some of the special functions of applied mathematics and physics, including Bessel functions and Legendre polynomials. Sturm-Liouville theory. Fourier series. Partial differential equations (PDEs): separation of variables, with applications to Laplace's equation, the wave equation and the heat equation. Classification of second order PDEs. Use of Maple to graph and investigate special functions, and to sum series.

Reading

W.E. Boyce and R.C. DiPrima: *Elementary Differential Equations and Boundary Value Problems* (Wiley, 1986). (R)
E. Kreyszig: *Advanced Engineering Mathematics*, 8th Edition (Wiley, 1999). (R)
E. Kreyszig & E.J. Norminton: *Maple Computer Guide: a Self-contained Introduction for E. Kreyszig Advanced Engineering Mathematics*, (Wiley, 2000). (R)
M.R. Spiegel: *Advanced Mathematics for Engineers and Introduction for Scientists* (Schaum Outline, McGraw-Hill, 1980). (R)

Assessment

Assessment of course work (10%)
1.5 hour examination at end of Semester (90%)

Chaos: Theoretical & Numerical Methods MAS2106

Module Leader: **Prof C F Barenghi**

Credits:	10	Programme:	24 lectures + 12 problem classes/drop-ins/practicals
Semester:	1	Pre-requisites	MAS1001 and MAS1101
		Co-requisites:	None

Description

Many fascinating physical phenomena, our weather system for example, are intrinsically nonlinear. This course provides an introduction to nonlinear behaviour, principally through the study of differential equations. It will lay the foundations for the ideas of chaos, an exciting and recently discovered phenomenon, involving random behaviour emerging from perfectly deterministic systems. Key ideas can be illustrated by fairly simple sets of equations, for example, three coupled nonlinear first-order differential equations.

The computer programming necessary will also be covered. No prior programming experience is assumed, and the emphasis will be on learning in a practical context (specifically, in the modelling of chaotic systems).

Aims and Objectives

To present the basic ideas of dynamical systems and the onset of chaotic behaviour. Students will be able to determine the fixed points of dynamical systems and their stability, and interpret the results; they will have an understanding of analytical and numerical techniques used to study chaotic systems. To introduce programming in a practical computer language. Students will be able to write programs in Fortran 90, making use of all of the fundamental programming structures; more specifically, they will be able to write programs to study chaotic dynamical systems.

Syllabus

Phase plane analysis. Fixed points and their stability. Lyapunov and asymptotic stability. The existence of periodic solutions; limit cycles. The Poincare-Bendixson theorem. A discussion of chaos. The Lorentz system. The appearance of chaos in systems of differential equations.

Introduction to programming. Basic program structure and implementation (editing, compiling, executing). Simple data types (integer, real, complex, logical, and character). Arrays of data. Input and output. Branching structures ('if' constructs). Repeating structures ('do' > loops).
Program blocks (subroutines, functions).

Reading

Steven H. Strogatz: *Nonlinear dynamics and chaos,* (Westview 1994) (R)
J. Gleick: *Chaos: Making a New Science* (Minerva, 1997). (R)
P.G. Drazin: *Nonlinear Systems* (C.U.P., 1992). (R)
D.W. Jordan & P. Smith: *Nonlinear Ordinary Differential Equations* (O.U.P., 1983). (R)
W.S. Brainard, C.J. Goldberg and J.C. Adams: *Programmer's Guide to Fortran 90* (Springer, 1990). (R)
T.M.R. Ellis, I.R. Philips, T.M. Lahey: *Fortran 90 Programming* (Addison-Wesley, 1994). (R)

Assessment

Assessment of course work (10%).
Projects (10%).
1.5 hour examination at end of Semester (80%).

Applications of Mathematical Methods to Finance MAS2201

Module Leader: **Dr A Vdovina**

Credits: 10

Semester: 1

Programme: 22 lectures, 2 feedback sessions, 4 problem classes, 2 tutorials and 3 practicals

Pre-requisites MAS1001, MAS1101, MAS1202

Co-requisites: None

Description

This course will give an overview of how mathematics is used in financial services. It will begin by introducing various forms of interest. The basics of saving, bonds and borrowing will be introduced. Simple continuous modelling will be presented. Both theoretical and practical applications of several financial topics will be covered. It will not give you the secret of making a fortune, but it will be useful whenever finances are involved. Many problems will be solved using the mathematical algebra package Maple.

Aims and Objectives

To present an introduction to mathematical finance.

Students will develop an understanding of interest rates, amounts of annuity, mortgage payments, financial bonds, and the elements of options pricing. Students will be able to read and construct precise mathematical arguments in financial mathematics to perform simple continuous modelling and calculations relevant to some financial concepts.

Syllabus

Interest, interest rates, compound interest formula, annuities, amount of an annuity, sinking funds, modelling with continuously varying interest rates, present value analysis, rate of return, amortization, mortgage payments, financial bonds, options pricing.

Reading

A. Mizrahi and M Sullivan: *Mathematics: An Applied Approach*, 7th Edition (Wiley 2000) (R)
S. M. Ross: *An Elementary Introduction to Mathematical Finance*, 2nd Edition (Cambridge UP, 2003) (R)

Assessment

Assessment of coursework (20%)
Assessment of project (10%)
1.5 hour examination at end of semester (70%)

Number Systems and the Foundations of Analysis MAS2202

Module Leader: **Dr A J Duncan**

Additional Lecturer(s): **Dr M C White**

Credits: 20

Semester: 1 and 2

Programme: 43 lectures, 9 problem classes, 4 tutorials and 2 feedback sessions

Pre-requisites MAS1001
Co-requisites: None

Description
This module is an introduction to Pure Mathematics. The central theme is the notion of proof; we will consider proofs by induction and contradiction. The module also introduces two principal branches of pure mathematics: algebra via number systems, and analysis which depends on the notion of a limit. In both of these areas we shall be interested in making precise statements and deciding whether or not they are true. Often we believe things to be true because of numerical calculations by hand, or by computer. These methods are valuable because they suggest possible truths. However, we do not know if a plausible statement is true until it has been proved.

Aims and Objectives
To present the notion and techniques of proof, illustrated by basic results in algebra and analysis.

Students will be able to read and construct precise mathematical arguments, proofs and examples, in simple cases; they will gain an understanding of the natural, rational and real number systems, and of the definition and properties of limits.

Syllabus
Logic and techniques of proof. Number theory including the Division Algorithm and the Fundamental Theorem of Arithmetic. Modular arithmetic. The real number system. Continued fractions. Rational and irrational numbers. Review of inequalities. Bounded sets and least upper bounds. Sequences and their limits. Infinite series.

Reading
R.P. Burt & A. Chetwynd: *A Cascade of Numbers* (Arnold 1996). (B)
R. Haggarty: *Fundamentals of Mathematical Analysis* (Addison-Wesley, 1993). (B)
K.M. Heal, M.L. Hanson & K.M. Rickard: *Maple V Learning Guide* (Springer, 1996). (B)
K.E. Hirst: *Numbers, Sequences and Series* (Arnold, 1995). (B)
M. Liebeck: *A Concise Introduction to Pure Mathematics* (Chapman and Hall, 2000). (B)

Assessment
Assessment of course work (20%).
1.5 hour examination at the end of each semester (each 40%)

Sets and Functions MAS2211

Module Leader: **Dr Z A Lykova**

Additional Lecturer(s):

Credits:	10	Programme:	24 lectures + 12 problem classes/drop-ins
Semester:	1	Pre-requisites	MAS1001, MAS1202
		Co-requisites:	None

Description
The differential calculus was discovered 350 years ago and ever since has been the single most important mathematical tool for the study of nature. From the beginning, even as the calculus was being applied in science and technology, there was concern about the apparent paradoxes and confusion about the properties of differentiation. It took leading mathematicians 200 years to formulate precise definitions of continuity and differentiability and to prove their fundamental properties: this constitutes the subject matter of real analysis, which we study in this module. We shall also encounter the remarkable notion of counting infinite sets: some infinite sets are more numerous than others, a fact that has many consequences.

Aims and Objectives
To develop an understanding of the role and importance of definition, proof and rigour in mathematics, of the formal ideas of limits and of the formalism of sets and mappings.

Students will deepen and reinforce their understanding of the role and importance of definition, proof and rigour in mathematics and of the formalism of sets and mappings.

Students will be able to formulate definitions and statements of theorems precisely and will enhance their understanding of quantifiers. They will be able to use the basic notions of real analysis and cardinality.

Syllabus
Sets and mappings. Domain and range. Limits and continuity. Intermediate Value Theorem. Differentiabilty. Mean Value Theorem. Increasing and decreasing functions. Injective, surjective and inverse mappings. Count ability and uncountability. Cantor's diagonal argument. Equivalence relations.

Reading
V. Bryant: *Yet Another Introduction to Analysis* (C.U.P., 2001) (R)
R. Haggarty: *Fundamentals of Mathematical Analysis*, 2nd Edition (Addison-Wesley, 1993). (R)

Assessment
Assessment of coursework (10%)
1.5 hour examination at end of Semester (90%)

Linear Algebra and Analysis MAS2212

Module Leader: **Prof S E Rees**

Credits: 10

Semester: 2

Programme: 24 lectures + 12 problem classes/drop-ins/practicals

Pre-requisites MAS1001, MAS1202

Co-requisites: MAS2211

Description

Ideas about limits have already been discussed in previous modules. They have been seen in the context of sequences and series in MAS1202/2202 in Stage 1/2 and in the context of functions in MAS2211/3211 in Stage 2/3. We now extend these ideas to integrals and give a proper treatment of integration, explaining how the 'area under a curve' comes to be related to the 'opposite of differentiation'. In more mathematical terms, we shall see why the 'Fundamental Theorem of Calculus' should be true, i.e., that integration really is the opposite of differentiation in the appropriate setting.

We shall extend our ideas on series to power series $\sum_{n=0}^{\infty} u_n x^n$, in the process extending notions of Maclaurin series for functions, for example $\sum_{n=0}^{\infty} \frac{1}{n!} x^n$ representing e^x. We shall discuss the range of values of *x* for which a power series $\sum_{n=0}^{\infty} u_n x^n$ converges.

Vectors and matrices were introduced in the first year courses where they were manipulated without any great attention to the validity of the operations. Thus extension of results from three to an arbitrary number of variables has been justified by an 'and so on', or the like, argument. Here we shall be more precise. Vector spaces, which are defined via axioms, provide us with the ideas for studying matrices and other sets, whilst avoiding the complications of describing them explicitly.

Aims and Objectives

To further develop an understanding of the role and importance of definition, proof and rigour in mathematics in the context of analysis and linear algebra.

Students will deepen and reinforce their understanding of the role and importance of definition, proof and rigour in mathematics, the development of integration as a limiting process, and the formal ideas of linear algebra.

Students will be able to show that certain functions are uniformly continuous. Students will be able to determine in many cases whether or not a series converges; they will be able to find the radius of convergence of certain power series. Students will be able to show that certain functions are integrable. They will be able to use the basic notions of linear algebra.

Syllabus

Uniform continuity, Riemann integration, series, linear algebra.

Reading

V. Bryant: *Yet Another Introduction to Analysis* (C.U.P., 2001) (B)

Assessment

Assessment of coursework (10%)
1.5 hour examination at end of Semester (90%)

Algebra MAS2213

Module Leader: **Prof P Jorgensen**

Credits:	10	Programme:	24 lectures + 12 problem classes/drop-ins
Semester:	2	Pre-requisites	MAS1001 and MAS1202
		Co-requisites:	None

Description

Modern algebra deals with sets equipped with operations resembling some or all of addition, subtraction, multiplication and division. One example of such a system is a field. The concept of a field is a natural one since we have a familiar structure with two operations which mimic addition and multiplication of numbers. We study examples of finite fields and also examples of fields consisting of complex numbers. We consider polynomials defined over a field and means of determining reducibility or irreducibility in some cases. We find that polynomials over a field form a system known as a ring and we consider the possibility of factorisation in a ring. In particular we study rings consisting of complex numbers and see examples of such rings in which some numbers have more than one irreducible factorisation.

Aims and Objectives

To introduce the viewpoint of modern algebra by extending the work of earlier modules and to provide a basis for further study of algebra.

Students will develop an understanding of the concept of an algebraic system defined by axioms with particular application to fields and rings. They will understand the relationship between polynomials defined over a field and the field itself.

Students will be able to perform simple calculations over a field. They will be able to test polynomials for reducibility in certain cases and to factorise real or complex polynomials into irreducible factors. They will be able to factorise elements of complex number rings into irreducible factors. Students will be able to demonstrate an understanding of the axiomatic systems of fields and rings and of the possibility of non-uniqueness of factorisation in rings.

Syllabus

Field axioms and examples. Applications to the study of polynomials, their roots and their factorisation. Axioms for a ring and examples including polynomial rings, Z_n, matrix rings and complex number rings. Factorisation in rings.

Reading

R.B.J.T. Allenby: *Rings, Fields and Groups* (Hodder and Stoughton). (E)

Assessment

Assessment of coursework (10%)
1.5 hour examination at end of Semester (90%)

Enumeration and Combinatorics MAS2216

Module Leader: **Dr A Duncan**

Credits: 10

Semester: 1

Programme: 24 lectures + 12 problem classes/drop-in sessions/practicals

Pre-requisites MAS1001 and MAS1202

Co-requisites: None

Description

The module involves the study of problems involving a finite number of elements, objects or structures; enumeration and counting; relations and functions; algorithmic problems.

Aims and Objectives

To equip students with a range of basic tools and methods for analysing and solving discrete problems, useful in all branches of mathematics. To enable the students to apply these techniques to naturally occurring problems. To reinforce the students' ability to read, understand and develop mathematical proofs. Students will gain an understanding of the nature of discrete problems, particularly those involving enumeration and graph theory. They will understand how to apply the techniques of combinatorial mathematics to such problems. They will understand when it is appropriate to apply algorithmic methods and how to apply them.

Syllabus

Pigeonhole principle, permutations and cycles, binomial and multinomial coefficients, counting subsets, partitions. Stirling numbers, inclusion-exclusion principle, recursive formulae, generating functions. Graphs, isomorphism, subgraphs, connectedness, walks and paths. Trees, spanning trees. Shortest-path algorithm, min-cut/max-flow algorithms; travelling salesman problem. Planar graphs and Euler's formula.

Reading

M Bóna: *A Walk Through Combinatorics* (World Scientific, 2002). (B)
L. Lovasz, J. Pelikan and K. Vesztergombi: *Discrete Mathematics* (Springer, 2003). (B)
R J Wilson: *Introduction to Graph Theory* (Addison Wesley, 1996). (B)

Assessment

In Course Assessment (10%)
1.5 hour examination at end of semester (90 %)

Introduction to Probability and Statistics MAS2301

Module Leader: **Prof R Henderson**

Additional Lecturer(s): **Dr P S Ansell**

Credits: 20

Semester: 2

Programme: 43 lectures, 9 problem classes, 4 tutorials, 2 feedback sessions and 6 practicals

Pre-requisites MAS1001
Co-requisites: MAS2001 (if MAS1001 not taken)

Description

The first part of the course will cover the key concepts required for further study of probability and statistics. We begin with the fundamentals of probability theory. Using basic counting arguments, we will see why you are more likely to guess at random a 7-digit phone number correctly, than to get all 6 numbers on the National Lottery correct. We will then move on to probability distributions and investigate how they can be used to model uncertain quantities such as the response of cancer patients to a new treatment, and the demand for season tickets at Newcastle United.

The second part of the course will be mainly concerned with using probability theory and observed data together in order to estimate the properties of a population. For example, we will see how we can use information on the actual survival rates of a group of patients to make statements about survival rates of patients in general, and how confident we can be about the accuracy of such statements. Key statistical ideas will be introduced in the examination of these questions.

Aims and Objectives

To develop ideas and methods essential for the study of probability and statistics.

By the completion of the course, students will be familiar with ideas of statistical modelling, data analysis and interpretation. They will have learned to use the statistical package R. Students will have had an opportunity to develop their report writing skills through project work.

Syllabus

Introduction to random variation and probability: probability axioms. Counting arguments. Conditional probability and independence. Discrete probability models. Continuous probability models. Calculation and interpretation of mean and variance. Practical illustrations and calculation of probabilities using R. Introduction to Statistical Inference. Unbiased estimators. Properties of estimators. Sampling distributions. Introduction to Bayesian and likelihood inference.

Reading

D.S Moore: *The Basic practice of Statistics*, 4th Edition (Freeman, 2007) (R)
M.J.Sullivan: *Statistics; Informed Decision using Data*, 2nd Edition (Pearson, 2007) (R)

Assessment

Assessment of course work (20%)
Project work (10%)
2 x 1.5 hour examination at the end of semester (each 35%)

Introduction to Statistical Inference MAS2302

Module Leader: **Dr D Walshaw**

Credits: 10

Semester: 1

Programme: 24 lectures + 12 problem classes/drop-in sessions/practicals

Pre-requisites MAS1301

Co-requisites: MAS2304

Description

This module develops further the ideas of statistical inference for which the foundations were laid in MAS1301. Properties of estimators and hypothesis tests will be reviewed. The method of maximum likelihood is introduced as a general approach to (non-Bayesian) estimation. The properties of maximum likelihood estimators are explored. Many of the important properties apply asymptotically, that is in large samples, and this will be shown by simulation using R. The asymptotic properties allow the construction of approximate (large sample) hypothesis tests and confidence intervals and this will be shown. Finally, in certain important cases, such as the normal distribution, exact (small sample) results can be obtained and these give tests and confidence intervals for the means and variances of normal populations. The use of the chi-squared, t and F distributions in this will be explained. Exact inference about proportions using the binomial distribution will also be explained.

Aims and Objectives

To develop a knowledge and understanding of basic ideas of statistical inference. Students will be introduced to the method of maximum likelihood as a general approach to estimation. They will learn the main asymptotic properties of maximum likelihood estimators, gain insight into these through simulation and learn how to use these properties to construct hypothesis tests and confidence intervals. Students will gain an understanding of exact (small sample) methods of inference for one and two binomial samples and for one and two normal samples and learn how to use these methods, including the use of the chi-squared, t and F distributions.

Syllabus

Review of likelihood and simple Bayesian inference. Parameter estimation: bias, consistency, mean square error. Hypothesis tests: power, effect of sample size. Inference for one and two sample (binomial) proportions; inference for one and two normal random samples (using chi-squared, t and F distributions, with definitions by functions of random variables). Maximum likelihood estimator (single parameter case). Sufficiency. Score and information. Asymptotic distribution of maximum likelihood estimator; (demonstrated by simulation). Standard error, confidence intervals and hypothesis testing (Z test) based on asymptotic normality. Use of R.

Reading

H.S.Migon and D.Gamerman: *Statistical Inference: An Integrated Approach* (Arnold, 1999). (B)
I.Miller and M.Miller: *John E. Freund's Mathematical Statistics with Applications* (Prentice Hall, 2004). (B)
H.R. Neave: *Statistics Tables* (Routledge, 1999). (E)
P.Garthwaite, I.Joliffe and B.Jones: *Statistical Inference*, 2nd Edition (OUP, 2002). (B)
L. Wasserman: *All of Statistics* (Springer, 2004). (B)

Assessment

Assessment of course work (10%)
Test exercises (10%)
1.5 hour examination at end of Semester (80%)

Applied Probability MAS2303

Module Leader: **Dr P J Avery**

Credits: 10

Semester: 2

Programme: 24 lectures + 12 problem classes/drop-in sessions

Pre-requisites MAS1301

Co-requisites: MAS2304

Description

How can we model the size of a population as it changes over time? How can we predict the chance of a gambler winning given a certain strategy? On average how much will he win? Undoubtedly, significant uncertainty is a central feature of almost every real-life problem, and questions of this nature arise naturally in many applications ranging from economics and finance through to engineering, microbiology and genetics. It is precisely this element of uncertainty which makes questions like ``How should I invest in the stock market?'' challenging and interesting. In this course, we shall discover how certain probabilistic techniques can be used to model and analyse systems or phenomena that evolve randomly over time.

Aims and Objectives

To develop skills in probabilistic reasoning and to gain familiarity with some of the main techniques involved in the analysis of random systems. To lay foundations for further study of Probability and Statistics.

Students will gain knowledge of issues arising in the study of the simple random walk, the gambler's ruin problem and branching processes. Students will become familiar with the notion of Markov chains, will learn how to classify states and find the stationary distributions(s), and will gain some understanding of their probabilistic description.

Syllabus

Review of probability ideas: conditioning arguments. Random walks: solution of gambler's ruin problems. Inequalities and limit theorems (WLLN, CLT). Branching processes: probability of ultimate extinction. Markov chains: definition and examples, Chapman-Kolmogorov equations, classification of states, notions of transience and recurrence, stationary and finite time distributions.

Reading

G.R. Grimmet & D.R. Stirzaker: *Probability and Random Processes* (Oxford, 2001). (B)
S.M. Ross: *Introduction to Probability Models* (Academic Press, 1997). (B)

Assessment

Assessment of course work (10%)
In-course test (10%)
1.5 hour examination at end of Semester (80%)

Foundations of Probability MAS2304

Module Leader: **Dr P S Ansell**

Credits:	10	Programme:	24 lectures + 12 problem classes/drop-in sessions/practicals
Semester:	1		
		Pre-requisites	MAS1301
		Co-requisites:	None

Description
Probability is the branch of mathematics which helps us to describe, analyse and understand chance phenomena. While the development of competence in probability is an essential preparation for the study of modern statistics, probability is also an important object of study for pure mathematicians and plays a key role in many areas of applied mathematics. Perhaps the most remarkable thing we discover is that even random objects demonstrate regular patterns of behaviour which can helpfully be thought of as laws of probability.

Frequently we need to examine two or more variables at a time. Although we could study each random variable of interest separately, it may be more useful to study them jointly in order to discover relationships between them.

In the course we develop properties of probability distributions and present techniques which enable random variables to be transformed or combined. Important applications of probability are discussed and some remarkable general results derived.

Aims and Objectives
To acquire the mathematical and probabilistic skills necessary for the further study of statistics.
By the completion of the course, students will be familiar with a range of discrete and continuous probability laws, know how to compute moments of random variables and the distributions of transformed random variables. Students will have some knowledge of bivariate distributions, correlation and covariance, expectation, moment and probability generating functions.

Syllabus
Review of probability; conditional probability and independence; discrete and continuous random variables; simulation and probability integral transform; bivariate distributions; covariance and correlation; expectation; probability and moment generating functions. Illustrations will be carried out using the statistical package R.

Reading
G.M. Clarke & D. Cooke: *A Basic Course in Statistics*, 5th Edition (Arnold, 2004). (R)
J.H. McColl: *Probability* (Arnold, 1995). (R)
S.M. Ross: *A First Course in Probability*, (Prentice Hall, 2005). (R)
N.A. Weiss: *A Course in Probability*, (Addison Wesley, 2005). (R)

Assessment
Assessment of course work (10%)
Test exercises (10%)
1.5 hour examination at end of Semester (80%)

Statistical Inference MAS2305

Module Leader: **Prof J N S Matthews**

Credits: 10

Semester: 2

Programme: 24 lectures + 12 problem classes/drop-in sessions/practicals

Pre-requisites None
Co-requisites: MAS2302 and MAS2304

Description

This module extends ideas of statistical inference developed in MAS2302. In that module likelihood inference was developed for single parameter problems. In realistic applications there will usually be several unknown quantities to be estimated simultaneously and so we need to extend the methods to general p-parameter estimation and inference. Likelihood provides us with a powerful and very general method for these problems even when there are many unknowns to be estimated. Often a response variable Y is influenced by the value of another variable X, sometimes called a covariate – this is known as a regression problem. For instance reaction time to a stimulus might depend upon age. Two types of regression model will be introduced, in both cases assuming there is just one covariate. First the simple linear regression model for responses which can be assumed to have Normal distribution; and second the logistic regression model for binary responses. In both cases maximum likelihood estimators are considered. Of course estimates from a sample will not be exactly equal to the true values, and so for inference we use the information matrix to derive associated measures of uncertainty. These can be used to provide intervals within which the true values should lie, or perhaps to test whether the true values take certain pre-specified values. Methods for assessing the assumptions which underlie the model will be introduced.

Aims and Objectives

To develop a knowledge and understanding of likelihood inference for general problems, especially regression and including hypothesis testing and interval estimation. Students will be able to write down the likelihood for statistical models and understand the steps needed to derive estimators and their properties. They will be able to use the computing environment R to simulate data from statistical models and to perform the calculations required for likelihood inference. They will use likelihoods to compare nested models. Students will understand the assumptions underlying simple linear and logistic regression models and will use these in practical applications. They will be able to fit regression models in R and interpret the associated computer output.

Syllabus

Review of methods for multivariate random variables: joint distributions; marginal and conditional distributions; covariance and correlation matrices. Review of single parameter likelihood methods. Multiple parameter likelihoods and information matrices. Asymptotic properties of estimators. The delta method. Likelihood ratio tests. Simple linear regression: properties of estimators. Logistic regression. Use of R for calculation and simulation. Residuals and other diagnostic tools.

Reading

H.S.Migon and D.Gamerman: Statistical Inference: *An Integrated Approach* (Arnold, 1999). (B)
I. Miller and M. Miller: *John E. Freund's Mathematical Statistics with Applications* (Prentice Hall, 2004). (B)
H.R. Neave: *Statistics Tables* (Routledge, 1999). (E)
P.Garthwaite, I.Joliffe and B.Jones: *Statistical Inference*, 2nd Edition (OUP, 2002). (B)
L. Wasserman: *All of Statistics* (Springer, 2004). (B)

Assessment

Assessment of course work (10%). In-course test (10%)
1.5 hour examination at end of Semester (80%)