Summary

This paper develops a spatial general equilibrium analysis of an agri-environmental policy in a suburban context. We present a static monocentric model of an open city where agricultural bid-rents and agricultural amenities vary endogenously in space. Amenities are valued by households and are thus a factor of urban decentralisation. This leads us to focus on the spatial effects of agri-environmental policies promoting amenities. The model characterises a suburban mixed land-use area where households and farmers share space. We provide theoretical evidence that agri-environmental policies are not adopted uniformly by farmers and that they impact on several city features. We highlight that the funding of an agri-environmental policy through household income taxation can modify urbanisation patterns. We also discuss its distributional aspects.

Keywords: agricultural amenities; land development; agri-environmental policy; urban sprawl; leapfrog; monocentric model.

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1 Introduction

Similar to La Fontaine’s rats, urban and rural agriculture have little in common. Farming at the urban fringe is intensive in terms of non-land inputs such as labour, capital, equipment or fertilisers. Further away from the city, rural agriculture relies more on land. This Thünenian spatial organisation has been thoroughly described in the literature (Beckmann, 1972; Katzman, 1974; OECD, 2009).

If farming is more intensive closer to cities, this is generally because farmland is more expensive and is therefore substituted by non-land inputs. There are two reasons for this. The first is historical and dates back to von Thünen. In an economic space, where farmers transport their produce to the city, farmland rents decrease in line with transportation costs. In a neoclassical framework, land use and crop management are more intensive close to the city (Beckmann, 1972). The second, modern reason, is linked to urban growth dynamics. Land conversion expectations and development irreversibility generate a growth premium and an option value which decrease with distance from the city and make up the agricultural component (agricultural returns) of farmland prices (Capozza & Helsley, 1989, 1990). Empirical investigations in the U.S. and in Europe show that both of these factors explain the observed negative gradient of farmland prices away from the city (Plantinga et al., 2002; Cavailhès & Wavresky, 2003; Livanis et al., 2006; Wu & Lin, 2010).

Heimlich & Barnard (1992) in the U.S. and Cavailhès & Wavresky (2007) in France have shown that farming at the urban fringe consumes significantly more inputs per hectare than farming further away. Because both positive and negative agricultural amenities are considered to be joint products of agricultural production (Abler, 2004; Hodge, 2008), because people value agricultural externalities (see Bergstrom & Ready (2009) for a review) and because urban development occurs mostly on farmland we can expect urban sprawl and agricultural intensity to be deeply interconnected. Hence, the problems associated with agricultural externalities (e.g. odours, nutrient run-off, water pollution, loss of hedgerows, landscape modifications, etc.) and land use conflicts are likely to be more severe at the urban fringe.

Agricultural externalities have become a major policy concern throughout the last decades. Agri-environmental policies (AEPs) that aim to maximise the social value of agriculture now consume a large proportion of the public funding dedicated to farming in developed countries. Whilst varying in importance and design throughout the world, agri-environmental policies are focused on preserving farmland and conserving natural and cultural resources. In the U.S., the two largest components of AEPs in the Farm Bill concern land retirement programs, such as the Conservation Reserve Program (CRP) and working land programs, such as the Environmental Quality Incentives Program (EQIP). In 2004, the U.S. Federal government spent $1.8 and $0.9 billion, respectively, on these two policies (Claassen et al., 2008). In Europe, AEPs are contained in Pillar 2 (Rural Development) of the Common Agricultural Policy (CAP). In contrast to the U.S., land retirement programs are minor elements of European policy, and AEPs focus

2. see Baylis et al. (2008) and Wunder et al. (2008) for in-depth reviews of agri-environmental policies (AEPs) in different countries.
3. The CRP is targeted toward the protection of erodible lands. Other programs, like the Grasslands Reserve Program or the Farm and Ranch Lands Protection Program aim at maintaining environmentally sound or culturally valued agricultural activities.
more on maintaining agricultural activities in disfavoured areas. A large part of the CAP is also targeted at working land programs such as Agri-Environmental Schemes. The European Fund for Rural Development, which finances AEPs in Europe, was funded with 96.3 billion euros for the period 2007-2013; of which 8.9 billion euros have been spent on AEPs, during the first four years of the program (2007-2010).

Such features question the effects of these programs at the urban fringe. Because agri-environmental policies may locally increase land scarcity and provide the environmental services demanded, it is reasonable to think that they have side-effects and may induce development locally. Land-use (Quigley & Rosenthal, 2005) and environmental regulations (Kiel, 2005) have been demonstrated to potentially generate important spillovers. Quigley & Swoboda (2007) have shown that land designation programs, such as the Endangered Species Act, have impacts that extend beyond the areas protected. Because designated lands cannot be developed, residential growth occurs throughout the city. Urban rents increase outside the protected area, thereby modifying development patterns and having significant redistributive effects among residents and landowners. Empirical investigations concerning the side-effects of AEPs are rare, because causality is difficult to identify. However, there is evidence that farmlands under agri-environmental programs are less likely to be developed than others (Wu & Cho, 2007; Liu & Lynch, 2011) and that farmland amenities are significantly valued (e.g. Bastian et al., 2002). Using conjoint analysis, Roe et al. (2004) have shown that the amenity value of preserved farmland is high, relative to the additional transportation costs of being located near to open-space amenities. Their results support the idea that AEPs can induce ex-urban development. Irwin & Bockstael (2004) have shown that open-space preservation policies may induce the development of neighbouring parcels of land. Recently, Towe (2010) has measured these side-effects for the CRP. Using the propensity score matching method, he has shown that parcels treated (i.e. preserved) under the CRP have significant effects on surrounding parcels, doubling their probability of development, and thus modifying development patterns. In Europe, land preservation is mostly concealed within urban zoning policies. Using the same method, Geniaux & Napoléone (2011) have shown that, in France, agricultural designation areas have significant effects on development patterns. Communes, with non-building agricultural and natural areas experience higher levels of development and urban growth than others. Geniaux & Napoléone (2011) attribute their results to an amenity effect generated by farmland and the protection of natural areas.

Despite their importance, working land programs have received little attention with respect to their impacts on urban development patterns. However, agri-environmental payments may be significant enough to reduce development, or at least delay it for the duration of the contract. In exchange for a payment, farmers engage, on a voluntary

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4Regulation (European Commission) 1698/2005, art.39.
5see Imbens & Wooldridge (2009) for a review of the recent developments in econometric methods to identify causal program effects.
6As discussed above, the CAP is not oriented towards land retirement or development easements.
7The lowest administrative level in France.
8In 2011, the EQIP program accounted for 13.2 million acres (5.3 million hectares) under contract, with federal funding of $865 million (NRCS, USDA), to which should be added local government allowed funds. In the European Union, AEPs amounted to 46 million hectares and 14.6 billion euros inclusive of national funding, for the period 2007-2010 (Directorate General for Agriculture and Rural Development, European Commission).
9Conservation agri-environmental contracts vary in duration: from 10 to 15 years in the U.S., and at
basis, in environmentally sound crop management practices oriented toward minimising negative externalities and promoting farming amenities and public goods. These benefits are likely to increase residential demand locally. In addition, if land is retained in agricultural use, residential rents will increase elsewhere in the city. Hence, AEPs may also have distributional effects.

In this paper, we undertake a theoretical analysis of the effects of voluntary agri-environmental working land programs on urban development patterns and wealth redistribution. We build our approach on previous work by Wu & Plantinga (2003), Cavaillé et al. (2004), Wu (2006), Kovacs & Larson (2007) and Bento et al. (2011). In all these models, agricultural amenities do not depend on the intensity of farming. Wu & Plantinga (2003), Wu (2006) and Kovacs & Larson (2007) focus on urban sprawl patterns. They describe how exogenous park-like amenities affect sprawl patterns and household segregation. In Bento et al. (2011), which analyses the efficiency of several anti-sprawl policies, amenities are endogenous, in the sense that they depend on the total amount of agricultural land outside the city which is affected by urban development. The model developed by Cavaillé et al. (2004) is the most similar to ours. In their approach, farmers’ behaviour is similar to von Thünen, as in our model. Farming amenities depend on the proportion of land in agricultural use at any suburban location. For our purposes, we endogenise amenities and relate them to land development and farming intensity.

Our analysis takes place in a static and open monocentric city, where endogenous agricultural amenities depend on both farmers and household behaviour. We introduce a voluntary agri-environmental policy aimed at limiting farming intensity and funded by households. We make some restrictive assumptions to keep our model as simple as possible and focus on the following core mechanism relating to the stylised facts described in this introduction: (i) farmland values decrease with distance from the city and land is substituted for non-land inputs; (ii) a decreasing farming intensity gradient is generated, where agriculture provides more environmental services further away from the city; (iii) farmers adopt agri-environmental contracts on the basis of the opportunity cost of farmland and the degree of constraint imposed on non-land inputs; (iv) households take into account agricultural externalities when choosing where to live.

The remainder of this paper is organised as follows. In section 2, we present the model, discuss the adoption of the AEPs by farmers and describe the conditions for a spatial general equilibrium. Section 3 presents a numerical illustration with plausible parameters; while in section 4 we study the impact of AEPs on urban structure and welfare distribution across agents. Section 5 concludes the paper.

2 A theoretical model for location decision with endogenous amenities

This section describes our spatial model and its equilibrium. As our spatial configuration is circular and symmetric, we place ourselves on a transect with the central business district (CBD) located at its origin in 0. The distance between the CBD and any point on the real line \([0; +\infty]\) is \(x\). It is assumed that all non-agricultural employment is concentrated at the CBD. Land is either dedicated to agricultural or residential use in least 5 years in Europe.
accordance with the rents offered by these two activities. We assume that all land is owned by absentee landlords.

2.1 Farmers’ behaviour

Farmers use a level $k = K/L$ of non-land inputs per hectare; where $K$ represents non-land inputs and $L$ land, to produce a quantity $y$ of output per hectare. The agricultural production function is given by $f(k)$ and is assumed Cobb-Douglas: $y = f(k) = A k^\alpha$ with $A > 0$ and $0 < \alpha < 1$. The production function is increasing in $k$ and concave. Farmers transport the agricultural output $y$ to the CBD. Thus, farmers are willing to pay a maximum rent $r_a(x)$ for a parcel located in $x$, depending on its distance from the CBD where they sell their produce. Transportation costs are given by $t$ and are assumed to be proportional to $x$. Crops and input prices, respectively $p$ and $p_k$, are exogenously set on competitive markets. In the absence of any public policy, the farmer’s program is given by (1):

$$\max_k \pi(k, x) = (p - tx)Ak^\alpha - p_k k - r_a(x)$$

(1)

The demand function of non-land inputs without any restrictive measure is found by maximising the profit $\pi(k, x)$ with respect to $k$:

$$k^*(x) = \left(\frac{\alpha A (p - tx)}{p_k}\right)^{\frac{1}{1-\alpha}}$$

(2)

$k^*(x)$ is a monotonous decreasing function of distance to CBD. Close to cities, farms tend to be intensive, characterised by a relatively high ratio $k$. Further away from cities, farms become progressively more extensive, as $k$ decreases. This direct result of the model is consistent with many empirical observations (Katzman, 1974; Heimlich & Barnard, 1992; Cavailhès & Wavresky, 2007). Competition drives farmers’ profits to zero, so that we can derive their bid-rent function at any location $x$:

$$r_a^*(x) = (p - tx)\frac{1}{1-\alpha} A \left(\frac{\alpha A}{p_k}\right)^{\frac{\alpha}{1-\alpha}} (1 - \alpha)$$

(3)

From (3), we derive that $\partial r_a^*/\partial x < 0$, meaning that $r_a^*(x)$ is a decreasing function of distance. At the extensive margin of agriculture, $r_a^*(x)$ tends toward $r_a$ where farming is no longer under the influence of the city. The limit of the urban area $\bar{x}$ solves $r_a^*(x) = r_a$. Without loss of generality, we set $r_a$ exogenously. Under our approach, the negative farmers bid-rent gradient is due to the imposed thünenian behaviour. As discussed in the introduction, this spatial organization is correctly supported by empirical evidence. This is because, storage and agri-food facilities may be located at the city boundary, and because some farmers engage in direct sales strategies (as in Cavailhès et al. (2004)). However, we fully recognise that farmland prices are also higher around cities because of the capitalisation of a growth premium, corresponding to expected rent increases due to the potential for land conversion to residential use (Capozza & Helsley, 1989). Our static approach cannot render this element, but it is fully compatible with this reality and should also be interpreted in this way.

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10 We implicitly assume a constant returns production function $F(K, L)$.
2.2 Agri-environmental policy

We now introduce a stylised working land AEP. Our AEP consists of a voluntary contract between the city government and farmers, with the aim of reducing the use of non-land inputs $k$ down to a fixed non-land inputs target $\hat{k}$. In return, all participating farmers are granted a uniform subsidy $\sigma$ per hectare. They will decide whether or not to enter into a contract\(^{11}\) on the basis of their private opportunity cost. The proposed AEP is thus fully described by the couple $(\sigma, \hat{k})$. This AEP mimics the voluntary working land conservation programs described in Wunder et al. (2008) and Baylis et al. (2008).

Adoption by farmers is endogenous to the model and depends on their location which, in our case, fully describes their opportunity cost. Farmers’ adoption of AEPs has been well documented in the literature. It has been thought to depend on factors relating to contract design\(^{12}\) and farmer characteristics\(^{13}\). Surprisingly, AEP adoption reviews by Knowler & Bradshaw (2007) and Baumgart-Getz et al. (2012) suggest that most of these factors are mitigated descriptors of farmers’ adoption of agri-environmental policies. It is also noted that these two literature reviews have not identified studies where proximity to urban areas\(^{14}\) is a determinant factor in AEP adoption. Indeed, they are rare. However, such studies are rather conclusive. Lynch & Lovell (2003) have shown that the probability of participation in agricultural land preservation programs in Maryland is lower close to the city and increases further away. Van Huylenbroeck et al. (2005) have shown that Belgian farmers closer to Brussels are less likely to adopt agri-environmental measures than their rural counterparts. A similar result is obtained by Bertoni et al. (2011) in Italy. Within our model, farmers closer to the city have the highest opportunity cost, and are less likely to adopt the AEP. All farmers are eligible for the contract and the profit maximisation program for volunteering farmers can be rewritten as follows:

$$\max_k \pi(k, x) = (p - tx)A\hat{k}^\alpha - pk - r_a(x) + \sigma \quad \text{s.t.} \quad k(x) \leq \hat{k} \tag{4}$$

Recalling that $k(x)$ is a strictly decreasing function of distance, there is a critical distance beyond which all farmers already comply with the maximum level of non-land inputs $\hat{k}$ decided by local authorities. This critical distance is given by $k^*(\hat{x}') = \hat{k}$, from which we derive:

$$\hat{x}' = \frac{1}{t} \left( p - \frac{pk}{A^\alpha} \hat{k}^{-\alpha} \right) \tag{5}$$

Farms located between the city and distance $\hat{x}'$ are more intensive and would therefore have to lower their use of non-land inputs, and thus their level of agricultural output, if they wish to adopt the AEP. A rational farmer accepts the contract if the subsidy compensates him for his loss of profit. The condition for a farmer to participate can be written:

\(^{11}\)Because we are in a static framework, the contract has no duration.

\(^{12}\)Those factors relate to contract duration, payment offered, enforcement and control modalities, technical assistance provided, etc.

\(^{13}\)Such as age, education, land tenure, the importance of non-farm income, farm size, implication in farming networks, etc.

\(^{14}\)Note, however, that factors related to proximity to urban centres, such as capital, income, farming intensity are generally significant and supportive of our assumptions
\[
(p - tx) Ak^\alpha - pk^k - r_a \leq (p - tx) A^{\hat{k}} - pk^{\hat{k}} - r_a + \sigma
\]

There is a location \( \hat{x} \), from which farmers agree to participate in the program, although they have to change their farming practices. \( \hat{x} \) solves:

\[
A \left( \frac{\alpha A}{pk} \right)^{\frac{\alpha}{1 - \alpha}} (1 - \alpha) (p - t\hat{x})^{1 - \alpha} - A^{\hat{k}} (p - t\hat{x}) + pk^{\hat{k}} - \sigma = 0
\]

(6)

We assume that local authorities have perfect foresight\(^{15}\) in terms of farmers’ input use at each location. For efficiency purposes, only farmers who actually change their use of non-land inputs are granted the subsidy. Farmers located further than \( \hat{x}' \) are not granted any subsidy, although it would be interesting for them to adopt the contract since they already comply with the input restrictions. Farmers located between the city and \( \hat{x} \) do not adopt the measure, as the subsidy fails to compensate them for the loss of profit. Farmers located between \( \hat{x} \) and \( \hat{x}' \) agree to adopt the measure and are granted the subsidy (see Figure 1). The spatial distribution of farming intensity is then given by:

\[
k(x) = \begin{cases} \hat{k}, & \text{if } x \in [\hat{x}, \hat{x}'] \\ k^*(x), & \text{otherwise} \end{cases}
\]

(7)

As we see in Figure 1, choosing \( \hat{k} \) is equivalent to choosing the adoption distance \( \hat{x}' \). Similarly, choosing \( \sigma \) is equivalent to choosing the depth \( (\hat{x} - \hat{x}') \) of the regulated area.

(7) allows us to derive the farmers’ bid-rent function:

\[
r^*_a(x) = \begin{cases} (p - tx) A^{\hat{k}} - pk^{\hat{k}} + \sigma, & \text{if } x \in [\hat{x}, \hat{x}'] \\ (p - tx)^{\frac{1}{1-\alpha}} A^{\frac{\alpha}{1-\alpha}} (1 - \alpha), & \text{otherwise} \end{cases}
\]

(8)

2.3 Households’ location decisions in urban and suburban areas

Households’ utility function is determined by the usual trade-off between accessibility to the CBD, land consumption and amenities. Each household chooses a combination of residential space \( q_h \), location \( x \) and a numeraire non-housing good \( s \) to maximise its utility, subject to the budget constraint \( w = r(x) q_h + s + \tau x + \eta \); where \( w \) is the gross household income, \( \tau \) is the round-trip commuting cost per unit of distance, \( r(x) \) is the housing price at location \( x \), and \( \eta \) is a lump-sum tax applied to all households, which is designed to fund the agri-environmental program.

We retain a Cobb-Douglas utility function \( U(s, q_h, a(x)) = q_h^\beta s^{1-\beta} a(x)^\gamma \), where \( a(x) \) represents spatially varying endogenous amenities. Without loss of generality, we assume that households living inside the city enjoy a uniform level of urban amenities. We normalise our measure of amenities, such that urban amenities are equal to unity. Outside the city, amenities are a joint-production of farming and vary with housing development. Land-use is mixed and shared between farmers and peri-urban households. Amenities in the peri-urban area are described in section 2.4. The empirical literature shows that the value of open-space amenities rapidly decays with distance (Irwin & Bockstael, 2004;\(^{15}\) This means that there are no transaction costs relating to the policy. It also means that there are no information asymmetries between the contractors. Informational rents are widely believed to undermine the efficiency of AEPs (e.g. Latacz-Lohmann & Van Der Hamsvoort, 1997; Ozanne et al., 2001; Canton et al., 2009).
Cavailhès et al., 2009) to become insignificant beyond a few hundred metres. Hence, we also suppose that there are no spillovers and that amenities are consumed by households only within their residential location.

The household maximisation problem defines the optimal demand for housing space and non-housing goods at each location:

\[ s^*(x) = (1 - \beta)(w - \tau x - \eta) \]  
\[ q_h^*(x) = \frac{\beta(w - \tau x - \eta)}{r_u(x)} \]

The bid-rent function for households depends on their location. In accordance with our definition of urban and peri-urban amenities, we have two bid-rents. Let \( r_u(x) \) and \( r_p(x) \) be the urban and peri-urban housing price at \( x \) respectively:

\[ r_u^*(x) = \left[ \frac{\beta^\beta (1 - \beta)^{1-\beta}(w - \tau x - \eta)}{V} \right]^{\frac{1}{\beta}} \]  
\[ r_p^*(x) = \left[ \frac{\beta^\beta (1 - \beta)^{1-\beta}(w - \tau x - \eta)}{V} \right]^{\frac{1}{\beta}} a(x)^{\frac{\gamma}{\beta}} \]

These bid-rent functions describe households’ maximum willingness-to-pay for housing at location \( x \). In equilibrium, households are indifferent to where they locate because their equilibrium level of utility is the same at each location and exogenous from the perspective of a single open city. From (11), we observe that the urban bid-rent decreases with distance and equals zero at \( x = (w - \eta)/\tau \), as commuting costs increase.
The peri-urban bid-rent (12) distribution across space is more difficult to predict. It is not necessarily a strictly decreasing function of distance, as households may outbid farmers in remote locations where, although commuting costs are high, they can enjoy higher levels of amenities. This bid-rent is similar to that found elsewhere in monocentric models dealing with amenities (Brueckner et al., 1999; Wu & Plantinga, 2003; Wu, 2006).

2.4 Farming amenities

Within our context, farming amenities should be understood as the net balance of positive externalities (e.g. landscape quality, cultural heritage, biodiversity) and negative ones (e.g. pollution, nuisances). We assume that the level of agricultural amenities is inversely proportional to non-land inputs per hectare (i.e., to the ratio $k$). More extensive farms, located far from the city, provide a higher level of agricultural amenities. Their extensive crop management favours the joint-production of positive externalities and lowers negative externalities. Conversely, farms located close to the city are more intensive and thus provide a lower level of agricultural amenities. Farming amenities are defined by (13):

$$a(x) = \delta \frac{\Theta(x)}{k(x)}$$

(13)

where $\delta$ represents the capacity of a given type of agriculture to provide amenities and $\Theta(x)$ is the fraction of agricultural land, as opposed to residential land.

The introduction of $\delta$ allows us to assume that some types of farming are more inclined to provide amenities than others, at any given level of non-land input intensity. $\delta$ describes the degree of jointness between amenities and farming. $\delta$ is fixed because our model only relies on one type of farming. As shown by Beckmann (1972), our model could also accommodate several agricultural outputs. Note, however, that the uneven spatial distribution of climate, soil and agronomic conditions, and the existence of agglomeration externalities, favour some degree of farming homogeneity and specialisation at the city level. Within our model, agriculture such as single crop farming or soil-less livestock breeding are supposed to generate only a low level of amenities that are positively valued by households (low $\delta$). On the contrary, grasslands or fruit horticulture is supposed to provide a higher level of such amenities, regardless of the use of non-land inputs (higher $\delta$). The value of $\delta$ is likely to vary between cities and countries depending on prevalent farming types and cultural characteristics. For example, in Kentucky, horse farms with stone and plank fences, picturesque barns and grasslands are likely to be associated with a high $\delta$, as described by Ready et al. (1997). In Wyoming (Bastian et al., 2002), this would be the case for areas dominated by elk habitat and scenic grasslands; while in areas where farming is intensive and undiversified $\delta$ would be low. Fansold & Lilieholm (1999), McConnell et al. (2005) and Bergstrom & Ready (2009) provide insightful reviews of the values associated with farmland and farming.

Through $\Theta(x)$, the amenity function accounts for the negative effect of development. As households move to the peri-urban area, agricultural amenities are progressively destroyed. This negative externality between households who prefer low density locations has been documented by Irwin & Bockstael (2002) and Roe et al. (2004), among others. Within our model, they occur in equilibrium (see section 2.5 for details). As peri-urban households move in, $\Theta(x)$ decreases, lowering the level of agricultural amenities and thus
lowering the peri-urban bid-rent until both bid functions are equal, leading to a mixed land use pattern. When $\Theta(x) = 0$ there is no agriculture at all and, therefore, no farming amenities. This configuration is impossible, as without any agricultural amenities, households would have no incentive to move to the peri-urban area. When $\Theta(x) = 1$ all the land is in agricultural use.

2.5 Spatial equilibrium

After deriving farmer and household behavioural functions, we now characterise spatial equilibrium within the urban area. Housing prices are pushed upwards in desirable locations so that in equilibrium, no household wants to move. This condition is satisfied when housing prices are given by (11) and (12) and farmland rent by (3). Land is occupied by the highest bidder. The prevailing land rent at any location is given by (14).

$$r^*(x) = \max \left\{ r_a^*(x), r_p^*(x), r_a^*(x) \right\}$$  \hspace{1cm} (14)

Definition of the city

As defined earlier, the urban area boundary is $x$, the distance at which agriculture is exogenous. The city boundary $x \in [0, x]$ solves $r_u^*(x) = r_a^*(x)$. The city is represented by the set of locations $C = \{ x < x \mid r_u^*(x) > r_a^*(x) \}$. The city boundary $x$ is thus determined by the competition between urban households and farmers for land. Household density $\rho_u^*(x)$ within the city is defined by:

$$\rho_u^*(x) = \frac{D(x)}{q_h^*(x)}$$  \hspace{1cm} (15)

where $q_h^*(x)$ is the residential plot size (in $m^2$/households) and $D(x)$ is the quantity of housing per unit of land (residential plot size per hectare in $m^2$/ha). For simplicity, we suppose that $D(x)$ is exogenously given and equal to 1. This assumption allows us to avoid the introduction of a developer, as is the case in several other models, (e.g. Wu, 2006; Quigley & Swoboda, 2007; Bento et al., 2011). This assumption comes at no cost with respect to the interpretation of the model. Note however, that it precludes us from deriving changes to the housing density.

Definition of the peri-urban area

Within the peri-urban area, land use is neither exclusively agricultural nor residential. Land is shared between farmers and households. The mixed land-use, peri-urban area is defined by the set of locations $P = \{ x > x \mid r_p^*(x) = r_a^*(x) \}$. Two possible configurations arise:

- Urban extension : the peri-urban area develops directly on the city boundary $\pi$. In this case, there must exist $x_2$ such that for all $x \in [x, x_2]$, we have $r_p^*(x) = r_a^*(x)$. Beyond $x_2$, the land is exclusively in agricultural use and for all $x \in [x_2, x]$, we have $r_a^*(x) > r_p^*(x)$.

\footnote{The derivation of the existence conditions of all the following patterns is available from the authors upon request.}
• Leapfrog development: within our context, we define leapfrog as a pattern of land development that is disconnected from the city. If there exists \(x_1 < x_2 < x\) such that for \(x \in [\pi, x_1] \cup [x_2, x]\), we have \(r_a^*(x) > r_p^*(x)\), that is agricultural use only, and for \(x \in [x_1, x_2]\), we have \(r_p^*(x) = r_a^*(x)\) which characterises mixed land-use.

**Characteristics of the periurban area** In the mixed land-use area, farmers’ and households’ bid-rents are equal. We have \(r_a^*(x) = r_p^*(x)\). We define \(\Theta(x)\) as the fraction of agricultural land at location \(x\). It is equal to:

\[
\Theta^*(x) = \rho_u^*(x) L^*(x) = 1 - \rho_p^*(x) q_h^*(x) \tag{16}
\]

where \(\rho_u(x)\) and \(\rho_p(x)\) are, respectively, the farmer and household density in the peri-urban area, and \(L(x)\) and \(q_h^*(x)\) their respective space consumption. Constant returns to scale do not allow us to determine \(\rho_a^*(x)\) and \(L^*(x)\) separately. Thus, the number of farmers \(N_a^*\) is indeterminate. As \(r_p^*(x) = r_u^*(x)a(x)\) and \(r_p^*(x) = r_a^*(x)\) in the mixed land-use area, we derive the level of amenities within the peri-urban area in equilibrium:

\[
a^*(x) = \left( \frac{r_p^*(x)}{r_a^*(x)} \right)^{\frac{\beta}{\gamma}} \text{ at any } x \in [x_1, x_2] \tag{17}
\]

Combining (17) with (13) and (16), we obtain the equilibrium proportion of agricultural land and the residential density within the peri-urban area:

\[
\Theta^*(x) = \frac{k^*(x)}{\delta} a^*(x) \text{ at any } x \in [x_1, x_2] \tag{18}
\]

\[
\rho_p^*(x) = \frac{1}{q_h^*(x)} \left[ 1 - \frac{k^*(x)}{\delta} a^*(x) \right] \text{ at any } x \in [x_1, x_2] \tag{19}
\]

**Population in equilibrium** In an open city, the utility level is exogenous and identical in all cities such that households are free to migrate in or out without any cost. In equilibrium, the developed area must provide housing for all households, the number of which is endogenous to the model and given by:

\[
\int_0^\pi \rho_u^*(x) 2\pi x dx = N_u^* \tag{20}
\]

\[
\int_{x_1}^{x_2} \rho_p^*(x) 2\pi x dx = N_p^* \tag{21}
\]

**Budget in equilibrium** To fund the agri-environmental measures, local authorities collect a lump-sum tax \(\eta\) from all households. Budget equilibrium requires that we have:

\[
\eta^* \left( N_u^* + N_p^* \right) = \sigma S_a^* \tag{22}
\]

where \(S_a^*\) is the agricultural area contracted to the program, given by:

\[
S_a^* = 2\pi \int_\pi^{x_2} \Theta^*(x) x dx \tag{23}
\]
Spatial general equilibrium A spatial general equilibrium in our economy is the 14-uple \( \{r_u^*, r_p^*, k^*, s^*, q_h^*, \rho_u^*, \rho_p^*, a^*, \Theta^*, N_u^*, N_p^*, \eta^*, S_a^* \} \) defined by (8), (11), (12), (2), (9), (10), (15), (19), (17), (18), (20), (21), (22), (23) plus the boundaries \( \{x, x_1, x_2, \hat{x}, \hat{x}' \} \) defined in the text.

2.6 Partial effects of \( \sigma, \hat{k} \) and \( \eta \) on the equilibrium

The spatial general equilibrium depends on the agri-environmental policy parameters \( (\sigma, \hat{k}) \). Establishing the complete comparative statics of the model is cumbersome. We choose, therefore, to explore the interdependent effects in a numerical analysis. However, to facilitate the interpretation of these complex conjugate effects, we derive here the partial effects of \( \sigma \) and \( \hat{k} \) on the different variables of the model. First, \( \sigma \) involves not only the redrawing of agricultural area but also the lump-sum tax on households. At the budget equilibrium (22), it is easy to establish:

\[
\frac{\partial \eta^*}{\partial \sigma} = \frac{S_a^*}{N_u^* + N_p^*} > 0
\]  

(24)

This means that an ambitious agri-environmental policy requires an increase in the lump sum tax on urban and suburban households. In Table 1 we show the effects of \( (\sigma, k) \) and \( \eta \) on the different variables related to the city (a proof of Table 1 is given in Appendix A.1). According to (24), the effects of \( \sigma \) and \( \eta \) are similar. An increase in \( \sigma \) leads to a decrease in the urban rent \( (\partial r_u^*/\partial \sigma < 0) \), density \( (\partial \rho_u^*/\partial \sigma < 0) \) and non-housing consumption \( (\partial s^*/\partial \sigma < 0) \). The decrease in urban rents allows households to consume more housing \( (\partial q_h^*/\partial \sigma > 0) \). An increase in \( \sigma \) also implies the narrowing of the city \( (\partial x/\partial \sigma < 0) \) and the decline of its population \( (\partial N_u^*/\partial \sigma < 0) \). Within our open city, urban households migrate outside the city following the tax increase. The maximum non-land input level \( \hat{k} \) does not directly affect the behaviour of urban households but has an impact on the agricultural bid-rent within the adoption area \( [\hat{x}, \hat{x}'] \). Under specific conditions of \( (\sigma, \hat{k}) \) for which the adoption area is located at the urban fringe, \( \hat{k} \) can have a positive direct effect on \( x \): farmers being restricted in their use of non-land inputs, they lower their bid-rent function and are outbidded by urban residents. However, \( \hat{k} \) also has indirect effects on households which are not taken into account with the partial derivatives. For example, from (23), an increase in \( \hat{k} \) would change the limits of the regulated area consequently modifying the budget constraint (22) and the tax required to finance the AEP which, in turn, would affect the rest of the economy.

<table>
<thead>
<tr>
<th>( \sigma, \eta )</th>
<th>( r_u^* )</th>
<th>( \rho_u^* )</th>
<th>( N_u^* )</th>
<th>( q_h^* )</th>
<th>( s^* )</th>
<th>( \Theta^* )</th>
<th>( N_p^* )</th>
<th>( \eta^* )</th>
<th>( S_a^* )</th>
<th>( x \in [\hat{x}, \hat{x}'] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{k} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>otherwise</td>
<td></td>
</tr>
</tbody>
</table>

In the suburban area, the effects of \( \sigma \) depend on the location of the regulated area as shown in Table 2 (a proof of Table 2 is given in A.2). The effect of \( \sigma \) on the bid rent functions is positive inside the regulated area \( x \in [\hat{x}, \hat{x}'] \) \( (\partial r_u^*/\partial \sigma > 0, \partial r_p^*/\partial \sigma > 0) \)
and negative outside for the peri-urban bid-rent, neutral for the agricultural bid-rent. The peri-urban bid-rent increases because the area benefits from higher amenities \((\partial a^*/\partial \sigma > 0)\). The proportion of farmland within the mixed land-use area increases with \(\sigma\) \((\partial \Theta^* / \partial \sigma > 0)\), while suburban household density \((\partial \rho_p^* / \partial \sigma < 0)\) decreases. Outside \([\hat{x}, \hat{x}'\)\), amenities and the proportion of farmland increase through a decrease in their bid rents \((\partial r_p^*/\partial \sigma < 0)\).

### Table 2: Partial effects of \((\sigma, \hat{k})\) on the suburban variables

<table>
<thead>
<tr>
<th>(\sigma, \eta)</th>
<th>(r_a^*)</th>
<th>(r_p^*)</th>
<th>(a^*)</th>
<th>(\Theta^*)</th>
<th>(\rho_p^*)</th>
<th>(x_1)</th>
<th>(x_2)</th>
<th>(\hat{x})</th>
<th>(\hat{x}')</th>
</tr>
</thead>
<tbody>
<tr>
<td>if (x \in [\hat{x}, \hat{x}'])</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+/-</td>
<td>+/-</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>otherwise</td>
<td>0</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(\hat{k})</td>
<td>if (x \in [\hat{x}, \hat{x}'])</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+/-</td>
<td>+/-</td>
<td>-</td>
</tr>
<tr>
<td>otherwise</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Inside the regulated area, an increase in \(\hat{k}\) has a negative effect on bid rent functions \((\partial r_p^*/\partial \hat{k} < 0, \partial r_a^*/\partial \hat{k} < 0)\) and on amenities \((\partial a^*/\partial \hat{k} < 0)\).

However, it increases the density of the suburban area \((\partial \rho_p^*/\partial \hat{k} > 0)\) in line with a decrease in farmers’ land within the mixed land-use area \((\partial \Theta^*/\partial \sigma < 0)\).

The partial effects of \(\sigma\) and \(\hat{k}\) on the size of the peri-urban area, characterised by \((x_1, x_2)\) is more difficult to predict. In most cases, an increase in the level of subsidy tends to decrease the size of the peri-urban area \((\partial x_1 / \partial \sigma > 0\) and \(\partial x_2 / \partial \sigma < 0\)\), which can be easily explained by the tax applied to households, reducing their bid-rent; and a change in \(\hat{k}\) does not affect the size of the peri-urban area. However, under specific conditions on the location of the regulated area, the impact of \(\sigma\) and \(\hat{k}\) on the size of the mixed land-use area can be either positive or negative, depending on the relative weight of \(\sigma\) and \(\hat{k}\) on farmers and peri-urban bid-rents.

Finally, it is easy to establish that \(\hat{x}\) decreases, i.e. gets closer to the city, in response to an increase in \(\sigma\) and \(\hat{k}\). \(\hat{x}'\) does not depend on the level of subsidy, but reacts negatively after an increase in \(\hat{k}\), similarly to \(\hat{x}\).

To gain an intuitive understanding of the interactions between AEPs and the city variables, we need to take into account both the direct and indirect effects of the parameters of the model. In the following section, we illustrate these interactions with a numerical simulation of the model.

### 3 Numerical model

The model’s structural parameters are evaluated to generate a plausible medium-sized French city, using data from the French National Institute of Statistics and Economic Studies (INSEE)\(^\text{17}\) and the Farm Accountancy Data Network (FADN)\(^\text{18}\). Parameter values are presented in Table 3. According to the income survey conducted by the INSEE, between 2000 and 2009, the average income for French households was 33,384 euros.


Around a quarter of their total consumption is dedicated to expenditures on housing (25.6% in 2010). Commuting costs for households are provided by the French Internal Revenue Service, around 0.4€/km. Assuming there are 1.5 workers per household travelling back and forth to the CBD throughout the year, we set \( \tau = 400€/km/year \). Finally, in order to generate a realistic urban bid-rent at the CBD, we set the equilibrium utility value to \( V = 10,100 \). Given the lack of relevant data on household preferences for amenities, we arbitrarily set \( \gamma = 0.2 \).

The proportion of non-land costs per hectare reported in the FADN is approximately 90% of the total costs and gives an average level of charges \( p_k^* = 1,861€/ha \) per farm, and an average gross product \( pA^*k = 1,714€/ha \) per farm, in 2009. Combining this with the estimated proportion of non-land costs per hectare, we obtain a ratio between the output price and the non-land input price of 2.85. We assume that this ratio is constant for any average French farm. We set \( \alpha = 0.8, p_k = 1 \) and \( p = 2.62 \) in order to generate realistic values. Transportation costs for farmers are set to \( t = 0.02€/km/output \) unit, so that the radius of the total available land under the influence of the urban area is \( x = 91.6 \) km.

Table 3: Parameter values and signification

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w ) Income level</td>
<td>33 000 €</td>
</tr>
<tr>
<td>( \tau ) Commuting cost</td>
<td>400 €/km/year</td>
</tr>
<tr>
<td>( \beta ) Housing budget share</td>
<td>0.25</td>
</tr>
<tr>
<td>( V ) Equilibrium utility level</td>
<td>10,100</td>
</tr>
<tr>
<td>( \gamma ) Preference for amenities</td>
<td>0.2</td>
</tr>
<tr>
<td>( p ) Price of agricultural output</td>
<td>2.62 €/output unit</td>
</tr>
<tr>
<td>( p_k ) Price of non-land input</td>
<td>1 €/input unit</td>
</tr>
<tr>
<td>( \alpha ) Elasticity of production factor</td>
<td>0.8</td>
</tr>
<tr>
<td>( t ) Transport costs for farmers</td>
<td>0.02 €/km/output unit/year</td>
</tr>
<tr>
<td>( A ) Technical constant</td>
<td>1</td>
</tr>
<tr>
<td>( \delta ) Farming-amenity degree of jointness</td>
<td>Urban extension case: 24</td>
</tr>
<tr>
<td></td>
<td>Leapfrog case: 18</td>
</tr>
</tbody>
</table>

Development patterns are depicted in Figure 2. We represent the two possible sprawl patterns: urban extension and leapfrog development. The occurrence of these scenarios depends, to some extent, on the capacity of agriculture to provide amenities (\( \delta \)). Figure 2a illustrates a case where the mixed land-use area is disconnected from the city. This urbanisation pattern is called leapfrog development and occurs when \( \delta \) is relatively low (i.e. \( \delta = 18 \)). The mixed land-use area is disconnected from the city when surrounding agricultural activity has a low capacity to provide amenities. The second benchmark configuration depicted in Figure 2b is characterised by a peri-urban area located adjacent to the urban area. This urban extension development pattern occurs when farming has a higher degree of joint-production of amenities.

The main characteristics of both benchmark cities are summarised in Table 4. The size of the mixed land-use area and the number of peri-urban households are higher in the case of urban extension. This is due to household preferences for amenities (\( \gamma \)).
remaining equal in both cases. An increase in the capacity of agriculture to provide amenities strongly encourages households to move in.

Table 4: Characteristics of our two benchmark cities

<table>
<thead>
<tr>
<th>Description variable</th>
<th>Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Urban extension</td>
</tr>
<tr>
<td>$\delta$ Capacity for a farm to provide amenities</td>
<td>24</td>
</tr>
<tr>
<td>$\bar{r}$ Radius of the city (km)</td>
<td>13.61</td>
</tr>
<tr>
<td>$(x_2 - x_1)$ Size of the periurban area (km)</td>
<td>47.67</td>
</tr>
<tr>
<td>$N_u$ Number of urban households</td>
<td>240,505</td>
</tr>
<tr>
<td>$N_p$ Number of periurban households</td>
<td>254,527</td>
</tr>
<tr>
<td>$\bar{\Theta}^*$ Mean fraction of agricultural land within the periurban area (%)</td>
<td>0.88</td>
</tr>
</tbody>
</table>

4 Effects of the agri-environmental schemes

Farms that are close to the city tend to be intensive and farmers are not necessarily interested in reducing their non-land inputs, unless they receive a high level of compensation. Therefore, the area included within agri-environmental schemes can be either more or less distant from the city’s boundaries. This depends on the level of payments to farmers ($\sigma$) and restrictions on non-land inputs ($\hat{k}$). Thus, there is a significant potential for agri-environmental policy to be involved in spatial organisation within nearby cities and to participate in the shaping of the suburban area. In this section, we study the effects of different combinations ($\sigma, \hat{k}$) on farmers’ decisions to participate in the agri-environmental scheme and the impact of these decisions on the urban structure.
4.1 Impact of farmers’ decision to participate

Using Eq.(6), we determine the effects of different combinations \((\sigma, \hat{k})\) on \(\hat{x}\). Figure 3a and 3b show the impact of AEPs on the total contracted area, for both benchmark configurations: leapfrog and urban extension. As can be seen, the shapes are similar for both scenarios. Formally, by choosing \(\hat{k}\), local authorities implicitly choose the location of the adoption area. The size of the adoption area then depends on their choice of \(\sigma\).

![Figure 3: Effect of the AEP on the total contracted area \(S^*_a\)](image)

Intuitively, an increase in the maximum permitted level of non-land inputs \(\hat{k}\), that is, less constraints on farmers, leads to a lower \(\hat{x}\), meaning that more intensive farmers - located closer to the CBD - are likely to agree to the contract. Similarly, but to a lesser extent, an increase in the subsidy level \(\sigma\) leads to a decrease in \(\hat{x}\). Indeed, higher compensation encourages more intensive farmers to adopt the AEPs. Hence, the proportion of agricultural land under contract increases, as the subsidy gets higher and \(\hat{k}\) lower. This result is due to two distinct mechanisms. Firstly, higher larger compensation leads to a larger participating area \((\partial \hat{x}/\partial \sigma < 0)\). Secondly, the decreasing convex shape of \(k(x)\) with distance \((\partial k^2/\partial x < 0\) within \([0, \underline{\xi}]\)) induces that, for the same level of subsidy \(\sigma\), the contracted area will be smaller nearer to the city than further away (see Figure 1).

4.2 Impact on the urban structure

This section examines the effects of AEPs on the urban structure through some key description variables: location of the urban boundary and on the size of the periurban area.

**Urban boundary** \((\underline{\tau})\)  The effects of the AEPs on the size of the urban area are depicted in Figure 4a and 4b for the urban extension and leapfrog cases, respectively. As urban households do not value agricultural amenities, their bid-rent function remains unaffected by a change in the supply of amenities. However, as urban households are taxed in order to fund the program, an increase in the level of subsidy leads to a decrease
in their urban bid-rent \( \partial r^*_p/\partial \eta < 0 \). Consequently, as the participating area increases in size (i.e. low \( \hat{k} \) and high \( \sigma \)), the location of the urban boundary tends to decrease, which is, indeed, what we observe in Figure 4a and 4b for approximatively \( \sigma > 100 \) €/ha and \( \hat{k} < 5 \) non-land input units. In addition, farmers who agree to participate in the AEP capitalise the subsidy in their bid-rent function, so that their bid-rent function increases. In cases where participating farmers are located at the urban fringe (i.e. high \( \hat{k} \) and high \( \sigma \)), farmers can outbid urban households. This explains the negative impact observed for \( \sigma \), varying from 200 to 300 €/ha, coupled with \( \hat{k} > 20 \) non-land input units.

![Figure 4: Effect of the AEP on x](image)

Figure 4: Effect of the AEP on \( x \)

It appears from this simulation that ambitious agri-environmental policy, funded by all inhabitants, could theoretically reduce the size of the city. In practice, this may lead to a real hindrance to urban development in both benchmark scenarios. Obviously, this result depends on the assumption of an open city, that allows both in- and out-migration, but it still shows the non-spatial neutrality of a public policy that supports the provision of a public good such as agricultural amenities.

**Size of the periurban area and development pattern**  The effect of AEPs on the size of the periurban area is more complicated. Figure 5a and 5b depict the variation in the size of the peri-urban area \( (x_2 - x_1) \) induced by various combinations of \( (\sigma, \hat{k}) \), compared to the benchmark cases. We consider these variations in the leapfrog and urban extension configurations, respectively. We observe two main antagonist effects following the introduction of AEPs. The first effect is the reduction in the size of the mixed land-use area, due to peri-urban households being taxed to fund the policy \( \partial r^*_p/\partial \eta < 0 \). The decrease in the peri-urban bid-rent, combined with an increase in the agricultural bid-rent, following the capitalisation of the subsidy, leads to a situation where farmers can outbid households. As a consequence, the peri-urban area tends to be reduced. However, the increased supply of amenities induced by the policy is an incentive for peri-urban households to move in \( \partial r^*_p/\partial a > 0 \). Depending on the location and magnitude of the adoption area, the negative tax effect can be compensated for by
the increased amenity level provided by farmers. Farmers located in the adoption area can therefore be outbid by peri-urban households, leading to a larger peri-urban area.

In the leapfrog case (Figure 5b), the negative tax effect is clearly visible at high levels of subsidy (σ > 150 €/ha) and intermediate and low levels of maximum non-land inputs use (k̂ < 10). These combinations (σ, k̂) ensure a large adoption area, leading to a high policy cost and therefore a higher tax applied to households. The negative effect prevails. However, we observe two distinct shapes, denoted A and B in Figure 5b, at k̂ between 12 and 15 and σ > 10 €/ha, and at k̂ < 3 and σ between 10 and 200 €/ha. In both cases, the adoption area is adjacent to the existing peri-urban area. In case A, the adoption area is situated next to the peri-urban area, but closer to the city. The level of amenities was originally too low to encourage households to relocate there. However, the adoption of AEPs has led to an increase in amenities causing households to outbid farmers, and thereby leading to the emergence of a second peri-urban belt (see Figure 6A). In the second case (B), the adoption area is situated just beyond the peri-urban area. The additional provision of amenities compensates households for their higher commuting costs so that they can afford to be situated further away from the city (see Figure 6B). These results highlight a potential undesirable effect resulting from the introduction of an AEP, i.e. an increase in the occurrence of leapfrog development.

Figure 5: Effect of the AEP on (x2 − x1) + (x4 − x3) in %

In the case of an urban extension city (Figure 5a), the variation in the peri-urban area resulting from different combinations of AEPs is more limited. It only varies from −0.6 to 0.8% (whereas the variation could range from −15 to 5% in the case of the leapfrog city). This difference is due to the prior existence of a larger peri-urban area in the case of the urban extension city, as opposed to the leapfrog city. The main positive variation in the size of the peri-urban area can be explained by the contraction of the urban area, leaving more space dedicated to mixed land-use. This happens for the previously identified combinations of (σ, k̂) which have a negative impact on π.
4.3 Welfare effects of the policy

In the following section we discuss the impact of different combinations of AEPs on land values, as a welfare criteria. We define the total land value for each agent as follows:

\[ R_u = 2\pi \int_0^\pi r_u^*(x)xdx \]  
\[ R_a = 2\pi \int_\Theta^\pi \Theta^*(x)r_a^*(x)xdx \]  
\[ R_p = 2\pi \int_{x_1}^{x_2} (1-\Theta^*(x)) r_p^*(x)xdx \]

We recall that \( \Theta^*(x) \) is the proportion of agricultural land. It varies between 0 and 1 within the peri-urban area, and is equal to 1 within an exclusively agricultural area. The total land value is defined by:

\[ R = R_u + R_a + R_p \]

Table 5 shows the impact of four different combinations of \( (\sigma, \hat{k}) \) on the total land value of our space, for a leapfrog and an urban extension benchmark configuration, respectively.

We observe that the impact of AEPs on the total land value \( R \) is negative overall, although it does not vary greatly. The introduction of any AEP combination lowers the global welfare level, compared to a situation where there is no policy in place. The largest decrease in total land value occurs under scenario where the regulated area is the largest, and thus the tax \( \eta \) is higher. This also corresponds to cases where the city decreases in size (i.e. high \( \sigma \) combined with either high enough \( \hat{k} \) or low \( \hat{k} \)). This is explained by the fact that \( R \) only involves \( r_u \) and \( r_a \), without accounting for \( r_p \). Therefore, our measurement of welfare is neutral to the proportion of residential use within the peri-urban area.
Table 5: Welfare effects of four AEP scenarios

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Benchmark</th>
<th>Agri-environmental scenarios</th>
</tr>
</thead>
<tbody>
<tr>
<td>(σ, ˆk)</td>
<td>(0,0)</td>
<td>(50, 5) (50, 22) (250, 5) (250, 22)</td>
</tr>
<tr>
<td>η</td>
<td>-</td>
<td>2.80 0.94 68.00 10.39</td>
</tr>
<tr>
<td>S_a^*</td>
<td>-</td>
<td>277 93 578 202</td>
</tr>
<tr>
<td>Ru</td>
<td>4,441</td>
<td>4,427 4,437 4,086 4,312</td>
</tr>
<tr>
<td>Rp for x ∈ [ ˆx, ˆx′]</td>
<td>- 256</td>
<td>38 311 56</td>
</tr>
<tr>
<td>Rp otherwise</td>
<td>5,176</td>
<td>4,927 5,147 4,704 5,106</td>
</tr>
<tr>
<td>Ra for x ∈ [ ˆx, ˆx′]</td>
<td>- 364 526</td>
<td>526 755 1,176</td>
</tr>
<tr>
<td>Ra otherwise</td>
<td>16,547</td>
<td>16,190 16,016 16,299 15,510</td>
</tr>
<tr>
<td>R</td>
<td>26,164</td>
<td>26,162 26,164 26,155 26,161</td>
</tr>
<tr>
<td>∆Ru in %</td>
<td>-</td>
<td>-0.31 -0.091 -3.52 -2.89</td>
</tr>
<tr>
<td>∆Rp in %</td>
<td>-</td>
<td>0.14 0.17 -0.57 -0.26</td>
</tr>
<tr>
<td>∆Ra in %</td>
<td>-</td>
<td>0.042 -0.028 1.12 0.84</td>
</tr>
<tr>
<td>∆R in %</td>
<td>-</td>
<td>-0.006 -0.0007 -0.07 -0.01</td>
</tr>
</tbody>
</table>

Leapfrog case

| (σ, ˆk)            | (0,0)     | (50, 5) (50, 22) (250, 5) (250, 22) |
| η                  | 0         | 6.05 1.63 69.45 17.96            |
| S_a^*              | -         | 370 100 780 212                 |
| Ru                 | 4,441     | 4,410 4,433 3,560 4,278         |
| Rp for x ∈ [ ˆx, ˆx′] | - 134    | 0 85 0              |
| Rp otherwise       | 1,414     | 1,284 1,409 904 1,358          |
| Ra for x ∈ [ ˆx, ˆx′] | - 486 564 | 564 980 1,232             |
| Ra otherwise       | 20,310    | 19,847 19,758 20,571 19,289   |
| R                  | 26,164    | 26,161 26,164 26,101 26,156   |
| ∆Ru in %           | -         | -0.70 -0.17 -8.15 -3.67       |
| ∆Rp in %           | -         | 0.31 -0.36 -7.32 -3.93        |
| ∆Ra in %           | -         | 0.11 0.06 2.18 1.04           |
| ∆R in %            | -         | -0.014 -0.002 -0.15 -0.029    |

Note: σ in €/ha, η and R in €, and S_a^* in km²
However, the welfare impact of the AEP can vary depending on the considered agent ($R_u$, $R_p$, $R_a$ in Table 5), meaning that significant redistributive effects can be observed between agents. We observe that urban households are systematically penalised by the introduction of the AEP. $R_u$ continues to decrease under any scenario, meaning that urban households help to fund a policy from which they gain no benefit. The negative effect of $\sigma$ on $R_u$ is detailed in Appendix A.3. It is consistent with the effects observed in Table 1, i.e. $\partial r^*_u/\partial \sigma < 0$ and $\partial \pi/\partial \sigma < 0$

With respect to peri-urban households, we observe that the impact of the AEP on $R_p$ can be either positive or negative. We explain this as the result of two conflicting mechanisms. First, peri-urban households are taxed, which has the direct effect of lowering their welfare, similarly to urban ones ($\partial r^*_p/\partial \sigma < 0$, $\partial x_1/\partial \sigma > 0$, $\partial x_2/\partial \sigma < 0$ if $x \notin [\hat{x}, \hat{x}']$, from Table 2). Secondly, the AEP enhances the production of agricultural amenities, which are valued by peri-urban households, and therefore automatically increases their bid-rent function ($\partial r^*_p/\partial \sigma > 0$ if $x \in [\hat{x}, \hat{x}']$, from Table 2). If peri-urban households are located within the regulated area $[\hat{x}, \hat{x}']$, they benefit from the introduction of the AEP. However, if they are situated outside the regulated area, their welfare level tends to decrease (see detailed calculations in Appendix A.3). Therefore, the total effect of the AEP on $R_p$ depends on the relative weight of these two mechanisms. For instance, we observe that, in the case of a low tax and accessible regulated area, the global impact of the AEP on the welfare of peri-urban households is positive, while under scenarios for which the tax is the highest, or the regulated zone is too far from the CBD, the global impact is negative.

Finally, we highlight that the welfare impact of the AEP on farmers is positive overall (see detailed calculations in Appendix A.3). This is due to the fact that, within the regulated area, farmers are granted a subsidy. Obviously, the larger the regulated area is, the greater the positive effect on $R_a$ will be. Moreover, even non-subsidised farmers benefit from the introduction of the AEP, in the sense that the tax impact on households makes them lower their bid-function, leaving a more important proportion of the land for agriculture ($\partial \Theta^*/\partial \sigma > 0$, from Table 2). However, in the urban extension case, for $\sigma = 50\text{€/ha}$ and $\hat{k} = 22$, the impact on $R_a$ is negative. This can be explained by the negative effect of $\hat{k}$ on $\pi$ (see Table 1), in the specific case where the regulated area is located right at the urban fringe. In this case, farmers who are restricted in their use of non-land inputs and granted a low level of subsidy, lower their bid-rent function and are then outbid by urban residents.

5 Conclusion

In this paper we have identified the spatial effects of a voluntary-based agri-environmental policy in the context of suburbanisation. Noting that the presence of natural amenities is a strong driver for urban sprawl, we modelled a monocentric city where amenities are generated by farmers whose behaviour is endogenised. Our model allows us to better understand the potential connection between spatially varying amenities and the location decision of households, particularly in the case where a public policy is introduced encouraging farmers to produce amenities.

Depending on the characteristics of the AEPs, and on the extent of their adoption by farmers, we identify several effects on urbanisation patterns. The taxation of households, in order to fund the policy, leads to a decrease in household bid-rents, which, in our open
city model, means a smaller city and smaller peri-urban area. However, depending on the location of the regulated area, we identify an undesirable side-effect of the policy, this being the emphasis on, and emergence of, leapfrog development. Indeed, if the measure is adopted to a large extent and within an accessible area, the level of agricultural amenities provided by agriculture increases, and becomes an incentive for households when making location decisions and thus encourages development where permitted. The net effect of agri-environmental measures on urban sprawl therefore depends on the negative effect of taxation and the positive effect of increased amenity.

Our analysis of the welfare impacts of the introduction of an AEP within our model allows us to identify various redistributive effects for each agent. We observe that the introduction of an AEP has a negative impact on the global welfare. Indeed, urban households are taxed to fund the policy but gain no benefit from it. The same remark stands for peri-urban households leaving in a non-regulated area. However, the impact on welfare can be positive for farmers, through the subsidy level and reduction in land competition; but also for peri-urban households living in the regulated area, through the increased amenity level.

We thereby highlight the fact that an agricultural policy is not neutral with regard to the competition for land use. Within a given territory, agricultural and land-use policies should take account of these relationships and be based on a more holistic approach that better reflects these interdependencies.

We leave for further research the transposition to a dynamic model, particularly suitable for an inter-temporal choice process such as urban sprawl. Furthermore, this would also allow the introduction of the irreversibility aspect of land development.

References


A  Appendixes - intended for reviewers only

A.1  Proof of Table 1

The first step is computation of the derivatives of \( r_u^* \), \( \rho_u^* \), \( N_u \), \( q^*_h \), \( s^* \) and \( \pi \) with respect to \( \sigma \) and \( \tilde{k} \). The signs of all partial derivatives are summarised in Table 1.

Using Eq(11), it follows that
\[ \frac{\partial r_u(\eta(\sigma))}{\partial \sigma} = \frac{\partial r_u}{\partial \eta} \frac{\partial \eta}{\partial \sigma} = -\frac{1}{\beta} \left( \frac{r_u^*}{w - \tau x - \eta(\sigma)} \right) \frac{S_a^*}{N_u^* + N_p^*} < 0 \] (29)

and since \( r_u \) does not depend on \( \hat{k} \), we have \( \partial r_u / \partial \hat{k} = 0 \).

Using Eq.(9) we can deduce
\[ \frac{\partial s(x, \sigma)}{\partial \sigma} = \frac{\partial s}{\partial \eta} \frac{\partial \eta}{\partial \sigma} = -(1 - \beta) \frac{\partial \eta}{\partial \sigma} < 0 \]

and since \( s(x) \) does not depend on \( \hat{k} \), we have \( \partial s(x) / \partial \hat{k} = 0 \).

Differentiating Eq.(10), with respect to \( \sigma \), and using Eqs. (24) and (29), yields (since \( \beta < 1 \))
\[ \frac{dq_k^*(x)}{\partial \sigma} = \frac{\partial q_k^*}{\partial \eta} \frac{\partial \eta}{\partial \sigma} + \frac{\partial q_k^*}{\partial r_u} \frac{\partial r_u}{\partial \sigma} = \frac{1}{r_u} \frac{S_a^*}{N_u^* + N_p^*} [1 - \beta] > 0 \] (30)

and since \( q_k^* \) does not depend on \( \hat{k} \), we have \( \partial q_k^* / \partial \hat{k} = 0 \).

Using Eq.(15) and Eq.(30), it follows
\[ \frac{\partial \rho_k^*(x, \sigma)}{\partial \sigma} = -\frac{D(x) \frac{dq_k^*(x)}{\partial \sigma}}{[q_k^*(x)]^2} < 0 \] (31)

and since \( \rho_k^* \) does not depend on \( \hat{k} \), we have \( \partial \rho_k^* / \partial \hat{k} = 0 \).

To compute \( \partial x / \partial \sigma \), we apply the implicit functions theorem to the equilibrium condition \( r_u^*(\pi) = r_a^*(\pi) \). We have:
\[ \frac{\partial x}{\partial \sigma} = -\frac{\partial r_u}{\partial \sigma} - \frac{\partial r_a}{\partial \sigma} \frac{\partial r_u}{\partial x} \frac{\partial x}{\partial \sigma} - \frac{\partial r_a}{\partial x} \frac{\partial x}{\partial \sigma} \]

Using Eq.(8), we obtain
\[ \frac{\partial r_a^*(x)}{\partial \sigma} = \begin{cases} 1 & \text{if } x \in [\hat{x}, \hat{x}'] \\ 0 & \text{otherwise} \end{cases} \] (32)

If \( \pi < \hat{x} \), we have \( \partial r_a^*(x) / \partial \sigma = 0 \). As \( \partial r_u / \partial \sigma < 0 \) and \( \partial r_u / \partial \pi < \partial r_a / \partial \pi \), we can conclude that \( \partial \pi / \partial \sigma < 0 \).

If \( \pi = \hat{x} \), we have \( \partial r_a^*(x) / \partial \sigma = 1 \). As \( \partial r_u / \partial \sigma < 0 \) and \( \partial r_a / \partial \pi < \partial r_a / \partial \pi \), we can conclude that \( \partial \pi / \partial \sigma < 0 \).

Similarly, to compute \( \partial \pi / \partial \hat{k} \), we apply the implicit function theorem to the equilibrium condition of \( \pi \)
\[ \frac{\partial \pi}{\partial \hat{k}} = -\frac{\partial r_u}{\partial \hat{k}} - \frac{\partial r_a}{\partial \hat{k}} \frac{\partial r_a}{\partial x} \frac{\partial x}{\partial \hat{k}} \frac{\partial x}{\partial \sigma} - \frac{\partial r_a}{\partial x} \frac{\partial x}{\partial \hat{k}} \frac{\partial x}{\partial \sigma} \]

As shown previously, we have \( \partial r_u / \partial \hat{k} = 0 \), and \( \partial r_u / \partial \pi < \partial r_a / \partial \pi \). \( \partial r_a / \partial \hat{k} \) is given using (8):
\[ \frac{\partial r_a^*(x)}{\partial \hat{k}} = \begin{cases} \alpha (p - tx) A \hat{k}^{\alpha - 1} - p_k & \text{if } x \in [\hat{x}, \hat{x}'] \\ 0 & \text{otherwise} \end{cases} \] (33)

If \( \pi < \hat{x} \), we have \( \partial r_a^*(x) / \partial \hat{k} = 0 \) and therefore, \( \partial \pi / \partial \hat{k} = 0 \).
If \( \pi = \hat{x} \), we have \( \partial r^*_a(x) / \partial \hat{k} < 0 \) and therefore, \( \partial \pi / \partial \hat{k} > 0 \).

To compute \( \partial N^*_u (\sigma) / \partial \sigma \), we differentiate Eq.(20)

\[
\frac{\partial N^*_u (\sigma)}{\partial \sigma} = \frac{\partial}{\partial \sigma} \left[ \int_0^{\pi(\sigma)} \rho^*_u (\sigma, x) 2\pi x \, dx \right] = \int_0^{\pi(\sigma)} \frac{\partial \rho^*_u (\sigma, x)}{\partial \sigma} 2\pi x \, dx + \frac{\partial \pi}{\partial \sigma} \rho^*_u (\sigma, x)
\]

As \( \partial \rho^*_u (\sigma, x) / \partial \sigma < 0 \) and \( \frac{\partial \pi}{\partial \sigma} < 0 \), we deduce that \( \partial N^*_u (\sigma) / \partial \sigma < 0 \).

### A.2 Proof of Table 2

Now we compute the partial derivatives of \( r^*_a \), \( r^*_p \), \( a^* \), \( \Theta^* \), \( \rho^*_p \), \( \hat{x} \), \( x_1 \) and \( x_2 \) with respect to \( \sigma \) and \( \hat{k} \). The signs of all partial derivatives are summarized in Table 2.

Using Eq.(8), we obtain

\[
\frac{\partial r^*_a(x)}{\partial \sigma} = \begin{cases} 
1 & \text{if } x \in [\tilde{x}, \tilde{x}'] \\
0 & \text{otherwise}
\end{cases}
\]

(34)

From Eq. 8, we also have

\[
\frac{\partial r^*_a(x)}{\partial \hat{k}} = \begin{cases} 
\alpha (p - tx) A \hat{k}^{a-1} - p_k < 0 & \text{if } x \in [\tilde{x}, \tilde{x}'] \\
0 & \text{otherwise}
\end{cases}
\]

Differentiating Eq.(17) with respect to \( \sigma \), yields

\[
\frac{d a^*(x, \sigma)}{d \sigma} = \frac{\partial a^*(x, \sigma)}{\partial r^*_a} \frac{\partial r^*_a}{\partial \sigma} + \frac{\partial a^*(x, \sigma)}{\partial r^*_u} \frac{\partial r^*_u}{\partial \sigma}
\]

\[
\frac{d a^*(x, \sigma)}{d \sigma} = \left[ \frac{1}{r^*_a \left( \frac{r^*_a}{r^*_u} \right)^{\frac{\beta}{\gamma}} - 1} \right] \frac{\partial r^*_a}{\partial \sigma} + \left[ - \frac{r^*_a(x, \sigma)}{r^*_u} \left( \frac{r^*_a(x, \sigma)}{r^*_u} \right)^{\frac{\beta}{\gamma} - 1} \right] \frac{\partial r^*_u}{\partial \sigma}
\]

If \( x \in [\tilde{x}, \tilde{x}'] \), we have \( \partial r^*_a / \partial \sigma = 1 \), then

\[
\frac{d a^*(x, \sigma)}{d \sigma} = \left\{ 1 - \frac{r^*_a(x, \sigma)}{r^*_u} \frac{\partial r^*_u}{\partial \sigma} \right\} \frac{1}{r^*_u} \left( \frac{r^*_a}{r^*_u} \right)^{\frac{\beta}{\gamma} - 1}
\]

(35)

As \( \partial r^*_u / \partial \sigma < 0 \), we deduce \( d a^*(x, \sigma) / d \sigma > 0 \).

If \( x \notin [\tilde{x}, \tilde{x}'] \), we have \( \frac{\partial r^*_u}{\partial \sigma} = 0 \), then

\[
\frac{d a^*(x, \sigma)}{d \sigma} = \left[ - \frac{r^*_a(x, \sigma)}{r^*_u} \left( \frac{r^*_a(x, \sigma)}{r^*_u} \right)^{\frac{\beta}{\gamma} - 1} \right] \frac{\partial r^*_u}{\partial \sigma}
\]

(36)

As \( \partial r^*_u / \partial \sigma < 0 \), we get \( d a^*(x, \sigma) / d \sigma > 0 \).

Differentiating Eq.(17) with respect to \( \hat{k} \), yields

\[
\frac{\partial a^*(x, \hat{k})}{\partial \hat{k}} = \beta \frac{1}{\gamma} r^*_a(x, \hat{k})^{\frac{\beta}{\gamma} - 1} \frac{\partial r^*_a}{\partial \hat{k}}
\]

If \( x \in [\tilde{x}, \tilde{x}'] \), we have \( \partial r^*_a / \partial \hat{k} < 0 \), so that \( \partial a^*(x, \hat{k}) / \partial \hat{k} < 0 \).
If \( x \notin [\bar{x}, \tilde{x}'] \), we have \( \frac{\partial x^*}{\partial x} = 0 \), so that \( \partial a^*(x, \hat{k})/\partial \hat{k} = 0 \).

Computation of the derivatives of \( \Theta^* \) with respect to \( \sigma \) and \( \hat{k} \), makes use of Eq.(18)

\[
\frac{d\Theta}{d\sigma} = a^* \frac{\partial k^*}{\partial \sigma} + k^* \frac{\partial a^*}{\partial \sigma}
\]

As \( \partial k^*/\partial \sigma = 0 \) and \( \partial a^*/\partial \sigma > 0 \), we get \( d\Theta/d\sigma > 0 \).

Similarly, as \( \partial k^*/\partial \hat{k} = 0 \), we obtain \( d\Theta^*/d\hat{k} < 0 \) (if \( x \in [\bar{x}, \tilde{x}'] \)) and \( d\Theta^*/d\hat{k} = 0 \) (otherwise).

Differentiating Eq.(19), with respect to \( \sigma \), yields

\[
\frac{\partial \rho^*_p(x)}{\partial \sigma} = -\frac{\partial q^*_h(x)}{\partial \sigma} \frac{1}{[q^*_h(x)]^2} \left[ 1 - \Theta^*(x, \sigma) \right] - \frac{1}{q^*_h(x)} \frac{\partial \Theta^*(x, \sigma)}{\partial \sigma}
\]

As \( \partial q^*_h(x)/\partial \sigma > 0 \) et \( \partial \Theta^*(x, \sigma)/\partial \sigma > 0 \) and \( \Theta^* < 1 \), we deduce that \( \partial \rho^*_p(x)/\partial \sigma < 0 \).

Differentiating Eq.(19), with respect to \( \hat{k} \), yields

\[
\frac{\partial \rho^*_p(x)}{\partial \hat{k}} = -\frac{\partial q^*_h(x)/\partial \hat{k}}{[q^*_h(x)]^2} \left[ 1 - \Theta^*(x, \hat{k}) \right] - \frac{1}{q^*_h(x)} \frac{\partial \Theta^*(x, \hat{k})}{\partial \hat{k}}
\]

As \( \partial q^*_h(x)/\partial \hat{k} = 0 \), we get

\[
\frac{\partial \rho^*_p(x)}{\partial \hat{k}} = -\frac{1}{q^*_h(x)} \frac{\partial \Theta^*(x, \hat{k})}{\partial \hat{k}}
\]

We obtain \( \partial \rho^*_p(x)/\partial \hat{k} > 0 \) (if \( x \in [\bar{x}, \tilde{x}'] \)) and \( \partial \rho^*_p(x)/\partial \hat{k} = 0 \) (otherwise).

The comparative statics on \( r^*_p \) may be obtained by differentiating Eq.(12) with respect to \( \sigma \). We have:

\[
\frac{dr^*_p(x, \sigma)}{d\sigma} = \frac{\partial r^*_p}{\partial \rho} \frac{\partial \rho}{\partial \sigma} + \frac{\partial r^*_p}{\partial a} \frac{\partial a}{\partial \sigma}
\]

By using Eqs. (11), (35) and (24) and (17) we obtain

\[
\frac{dr^*_p(x, \sigma)}{d\sigma} = a^* s^* r_u \frac{1}{r_a} + \frac{1}{\beta (w - \tau x - \eta(\sigma))} \frac{1}{N^*_u + N^*_p} \left\{ a^* s^* - 1 \right\} r_u \quad \quad (37)
\]

As \( a \geq 1 \), we get \( dr^*_p/\partial \sigma > 0 \).

Differentiating Eq.(12) with respect to \( \hat{k} \) leads to:

\[
\frac{dr^*_p(x, \sigma)}{d\hat{k}} = \frac{\partial r^*_p}{\partial \rho} \frac{\partial \rho}{\partial \hat{k}} + \frac{\partial r^*_p}{\partial a} \frac{\partial a}{\partial \hat{k}}
\]

As \( \partial \rho/\partial \hat{k} = 0 \), we have

\[
\frac{dr^*_p(x, \sigma)}{d\hat{k}} = \left[ \beta \left( 1 - \beta \right)^{1-\beta} \left( w - \tau \left( \frac{x - \eta(\hat{k})}{\hat{V}} \right) \right) \right]^{\frac{1}{\beta}} a(x, \hat{k})^{\frac{2}{\beta}} - 1 \frac{\partial a}{\partial \hat{k}}
\]

Using the partial derivatives of \( a(x) \), we deduce that:

\[
\frac{dr^*_p(x, \sigma)}{d\hat{k}} < 0 \quad \text{if} \quad x \in [\bar{x}, \tilde{x}']
\]

\[
\frac{dr^*_p(x, \sigma)}{d\hat{k}} = 0 \quad \text{otherwise}
\]
The effect of $\sigma$ and $\hat{k}$ on $\hat{x}$, can be obtained by differentiating Eq.(6) with respect to $\sigma$ and $\hat{k}$. Using the implicit function theorem, we obtain

$$\frac{\partial \hat{x}}{\partial \sigma} = -\left(\frac{-t_\alpha A}{1-\alpha} \left(\left(\frac{\alpha A}{p}\right)^{1-\alpha} (1-\alpha) (p-t\hat{x})^{\frac{\alpha}{1-\alpha}} + t A \hat{k}^\alpha\right)\right).$$

(38)

As $\hat{x} < \bar{x}$, we have $p > t\hat{x}$, which involves the denominator of (38) being negative. Thus, we deduce that $\partial \hat{x}/\partial \sigma < 0$. Similarly, we deduce that $\partial \hat{x}/\partial \hat{k} < 0$.

The effect on $\hat{x}'$, can be obtained by differentiating Eq.(5) with respect to $\sigma$ and $\hat{k}$. We get $\partial \hat{x}'/\partial \sigma = 0$ and $\partial \hat{x}'/\partial \hat{k} < 0$.

Finally, we compute the partial derivatives of $x_1$ and $x_2$ with respect to $\sigma$ and $\hat{k}$. Using the implicit function theorem on the equilibrium condition of the peri-urban area $r^*_p(x_1) = r^*_a(x_1)$ and $r^*_p(x_2) = r^*_a(x_2)$.

$$\frac{\partial x_1}{\partial \sigma} = -\frac{\partial r_p}{\partial x_1} \frac{\partial r_a}{\partial x_1}$$

As shown previously, when $x \notin [\hat{x}, \hat{x}']$, we have $\partial r_p/\partial \sigma < 0$ and $\partial r_a/\partial \sigma = 0$. Furthermore, we have $\partial r_p/\partial x_1 > \partial r_a/\partial x_1$. We can conclude that $\partial x_1/\partial \sigma > 0$. When $x \in [\hat{x}, \hat{x}']$, we have $\partial r_p/\partial \sigma > 0$ and $\partial r_a/\partial \sigma = 1$, so that we can not conclude on the sign of $\partial x_1/\partial \sigma$.

We proceed similarly for $x_2$. As $\partial r_p/\partial x_2 < \partial r_a/\partial x_2$, we cannot conclude on $\partial x_2/\partial \sigma$ for $x \in [\hat{x}, \hat{x}']$ but $\partial x_2/\partial \sigma < 0$ otherwise.

We now study the effects of $\hat{k}$:

$$\frac{\partial x_1}{\partial \hat{k}} = -\frac{\partial r_p}{\partial x_1} \frac{\partial r_a}{\partial x_1}$$

If $x \notin [\hat{x}, \hat{x}']$, we have $\partial x_1/\partial \hat{k} = 0$, and similarly $\partial x_2/\partial \hat{k} = 0$. However, when $x \in [\hat{x}, \hat{x}']$, we cannot conclude on the sign of the partial derivative. It depends on the relative weight of $\hat{k}$ on $r_p$ and $r_a$ respectively.

A.3 Decomposition of the welfare effects

In this appendix, we illustrate the decomposition of the welfare effects of the AEP. Because the calculations depend on the location of the regulated area, there are 6 possibilities\(^{19}\). To keep things understandable, let us consider only the case where the regulated area is fully comprised within the peri-urban area: $\bar{x} < x_1 < \hat{x} < \hat{x}' < x_2 < \bar{x}$. This is the predominant case, both for the urban extension and the leapfrog configurations. We focus only on the derivative of the total rent $R = R_u + R_p + R_a$ with respect to $\sigma$.

To restrict the length of the equation, we omit $(\sigma, \hat{k})$ in the functions arguments when it does not alter the comprehension.

$$R_u = 2\pi \int_0^{\pi(\sigma, \hat{k})} r_u(x, \sigma, \hat{k}) x dx$$

\(^{19}\) $\bar{x} < \hat{x} < \hat{x}' < x_1 < x_2 < \bar{x}$; $\bar{x} < \hat{x} < x_1 < \hat{x}' < x_2 < \bar{x}$; $\bar{x} < \hat{x} < x_1 < x_2 < \hat{x}' < \bar{x}$; $\bar{x} < x_1 < \hat{x} < \hat{x}' < x_2 < \bar{x}$; $\bar{x} < x_1 < x_2 < \hat{x} < \hat{x}' < \bar{x}$. 

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Moreover, note that for some given levels of $\sigma$ and $\hat{k}$, an additional leapfrog area $[x_3, x_4]$ can modify the rent beyond $\pi$. Differentiating (41), we have:

\[
\frac{\partial R_p}{\partial \sigma} = 2\pi \int_{x_1(\sigma, \hat{k})}^{x_2(\sigma, \hat{k})} (1 - \Theta(x, \sigma, \hat{k})) r_p(x, \sigma, \hat{k}) dx
\]

(41)

For various locations of the regulated area, the integrating limits should be switched. Moreover, note that for some given levels of $\sigma$ and $\hat{k}$, an additional leapfrog area $[x_3, x_4]$ can modify the rent beyond $\pi$. Differentiating (41), we have:

\[
\frac{\partial R_p}{\partial \sigma} = 2\pi \int_{x_1}^{\hat{x}} (1 - \Theta(x)) \frac{\partial r_p(x)}{\partial \sigma} dx - 2\pi \int_{\hat{x}}^{\bar{x}} \frac{\partial \Theta(x)}{\partial \sigma} r_p(x) dx + 2\pi \frac{\partial x}{\partial \sigma} \hat{x}(1 - \Theta(x)) r_p(\hat{x})
\]

(42)

\[
-2\pi \frac{\partial x_1}{\partial \sigma} (1 - \Theta(x)) r_p(x_1) + 2\pi \int_{\hat{x}}^{\bar{x}} (1 - \Theta(x)) \frac{\partial r_p(x)}{\partial \sigma} dx - 2\pi \int_{\hat{x}}^{\bar{x}} \frac{\partial \Theta(x)}{\partial \sigma} r_p(x) dx
\]

\[
+2\pi \frac{\partial \hat{x}}{\partial \sigma} \hat{x}'(1 - \Theta(x)) r_p(\hat{x}) - 2\pi \frac{\partial \bar{x}}{\partial \sigma} \bar{x}(1 - \Theta(x)) r_p(\bar{x}) + 2\pi \int_{\hat{x}}^{\bar{x}} (1 - \Theta(x)) \frac{\partial r_p(x)}{\partial \sigma} dx
\]

\[
-2\pi \int_{\hat{x}}^{x_2} \frac{\partial \Theta(x)}{\partial \sigma} r_p(x) dx + 2\pi \frac{\partial x_2}{\partial \sigma} x_2(1 - \Theta(x)) r_p(x_2) - 2\pi \frac{\partial \hat{x}}{\partial \sigma} \hat{x}'(1 - \Theta(x)) r_p(\hat{x}')
\]

After some simplifications and using Table 2, we can sign each element:

\[
\frac{\partial R_p}{\partial \sigma} = 2\pi \int_{x_1}^{\hat{x}} (1 - \Theta(x)) \frac{\partial r_p(x)}{\partial \sigma} dx - 2\pi \int_{x_1}^{\bar{x}} \frac{\partial \Theta(x)}{\partial \sigma} r_p(x) dx + 2\pi \frac{\partial x}{\partial \sigma} \hat{x}(1 - \Theta(x)) r_p(\hat{x})
\]

(43)

\[
+2\pi \int_{\hat{x}}^{x_2} (1 - \Theta(x)) \frac{\partial r_p(x)}{\partial \sigma} dx - 2\pi \int_{\hat{x}}^{x_2} \frac{\partial \Theta(x)}{\partial \sigma} r_p(x) dx + 2\pi \int_{\hat{x}}^{x_2} (1 - \Theta(x)) \frac{\partial r_p(x)}{\partial \sigma} dx
\]

\[
-2\pi \int_{\bar{x}}^{x_2} \frac{\partial \Theta(x)}{\partial \sigma} r_p(x) dx + 2\pi \frac{\partial x_2}{\partial \sigma} (1 - \Theta(x)) x_2 r_p(x_2) - 2\pi \frac{\partial \bar{x}}{\partial \sigma} (1 - \Theta(x)) x_1 r_p(x_1)
\]

We now spatially decompose the total effect on $R_p$. Between $x_1$ and $\hat{x}$, we have:

\[
\frac{\partial R_p}{\partial \sigma} \bigg|_{[x_1, \hat{x}]} = 2\pi \int_{x_1}^{\hat{x}} (1 - \Theta(x)) \frac{\partial r_p(x)}{\partial \sigma} dx - 2\pi \int_{x_1}^{\bar{x}} \frac{\partial \Theta(x)}{\partial \sigma} r_p(x) dx
\]

(44)

Between $\hat{x}$ and $x_2$, we have:

\[
\frac{\partial R_p}{\partial \sigma} \bigg|_{[\hat{x}, x_2]} = 2\pi \int_{\hat{x}}^{x_2} (1 - \Theta(x)) \frac{\partial r_p(x)}{\partial \sigma} dx - 2\pi \int_{\hat{x}}^{x_2} \frac{\partial \Theta(x)}{\partial \sigma} r_p(x) dx
\]

(45)
Outside the regulated area, the peri-urban household rent decreases through two channels. First, there is the negative effect of the tax. It is the first term in (44) and (45). There is also a density effect (the second term) because the number of households diminishes (their bid rent has decreased).

Inside the regulated area, between \( \hat{x} \) and \( \hat{x}' \), we have:

\[
\frac{\partial R_p}{\partial \sigma} \bigg|_{[\hat{x}, \hat{x}']} = 2\pi \int_{\hat{x}}^{\hat{x}'} \frac{(1 - \Theta(x)) \partial r_p(x)}{\partial \sigma} x dx - 2\pi \int_{\hat{x}}^{\hat{x}'} \frac{\partial \Theta(x)}{\partial \sigma} r_p(x) x dx
\]

(46)

The change in the total rent in this area is not easily signed. The bid-rent increases (due to the amenity improvement,) but the density effect is still negative. This is because households are taxed, while farmers are subsidised.

Moreover, we need to add the effect of the AEP on the limits of the leapfrog area. It is given by:

\[
\frac{\partial R_p}{\partial \sigma} \bigg|_{[x_1, x_2]} = 2\pi \frac{\partial x_2}{\partial \sigma} (1 - \Theta(x)) x_2 r_u(x_2) - 2\pi \frac{\partial x_1}{\partial \sigma} (1 - \Theta(x)) x_1 r_u(x_1)
\]

(47)

The diminishing peri-urban household bid-rent reduces the mixed land-use area \((x_2 - x_1)\). It has a negative effect on the total rent.

Finally, we turn to \( R_a \). Remembering that outside the mixed land-use area, \( \Theta(x) = 1 \) we have:

\[
R_a = 2\pi \int_{\hat{x}_1(\sigma, \hat{k})}^{\hat{x}_2(\sigma, \hat{k})} \Theta(x) r_a(x, \sigma, \hat{k}) x dx
\]

(48)

Recalling that outside the regulated area, \( r_a \) is not changed by the AEP, we have \( \frac{\partial r_a(x)}{\partial \sigma} = 0 \), differentiating (48) yields:

\[
\frac{\partial R_a}{\partial \sigma} = 2\pi \int_{\hat{x}_1}^{\hat{x}_2} \frac{\partial \Theta(x)}{\partial \sigma} r_a(x) x dx + 2\pi \frac{\partial \hat{x}}{\partial \sigma} \Theta(\hat{x}) r_a(\hat{x}) \hat{x} - 2\pi \frac{\partial x}{\partial \sigma} \Theta(x_1) r_a(x_1) x_1
\]

\[
+ 2\pi \frac{\partial \hat{x}}{\partial \sigma} \Theta(\hat{x}) r_a(\hat{x}) \hat{x}' - 2\pi \frac{\partial \hat{x}}{\partial \sigma} \Theta(\hat{x}) r_a(\hat{x}) \hat{x}
\]

\[
+ 2\pi \int_{\hat{x}_1}^{\hat{x}_2} \frac{\partial \Theta(x)}{\partial \sigma} r_a(x) x dx + 2\pi \frac{\partial \hat{x}}{\partial \sigma} \Theta(\hat{x}) r_a(\hat{x}) \hat{x}' - 2\pi \frac{\partial \hat{x}}{\partial \sigma} \Theta(\hat{x}) r_a(\hat{x}) \hat{x}'
\]

(49)
which can be simplified and signed as follows:

$$\frac{\partial R_a}{\partial \sigma} = 2\pi \int_{x_1}^{x} \frac{\partial \Theta(x)}{\partial \sigma} r_a(x) dx + 2\pi \int_{\tilde{x}}^{\hat{x}} \frac{\partial \Theta(x)}{\partial \sigma} r_a(x) dx + 2\pi \int_{\hat{x}}^{x} \frac{\Theta(x)}{\partial \sigma} \frac{\partial r_a(x)}{\partial \sigma} dx \quad (50)$$

$$+ 2\pi \int_{x'}^{x_2} \frac{\partial \Theta(x)}{\partial \sigma} r_a(x) dx + 2\pi \frac{\partial x_2}{\partial \sigma} \Theta(x_2) r_a(x_2) x_2 - 2\pi \frac{\partial x_1}{\partial \sigma} \Theta(x_1) r_a(x_1) x_1$$

Hence, we can spatially decompose the welfare effects for farmers. Outside the regulated area, i.e. for $x \in [\pi, \hat{x}] \cup [\hat{x}', \pi]$, we have:

$$\frac{\partial R_a}{\partial \sigma} \bigg|_{[\pi, \hat{x}] \cup [\hat{x}', \pi]} = 2\pi \int_{x_1}^{\hat{x}} \frac{\partial \Theta(x)}{\partial \sigma} r_a(x) dx + 2\pi \int_{\hat{x}}^{x_2} \frac{\partial \Theta(x)}{\partial \sigma} r_a(x) dx \quad (51)$$

$$+ 2\pi \frac{\partial x_2}{\partial \sigma} \Theta(x_2) r_a(x_2) x_2 - 2\pi \frac{\partial x_1}{\partial \sigma} \Theta(x_1) r_a(x_1) x_1$$

The AEP has an effect on the agricultural rent outside the regulated area while the farmers bid-rent function remains identical. This is due to the taxation effect on households which decreases their bid-rent hence increasing the density of farmers in the mixed land-use area. This is the effect described in (51). This effect adds to the contraction of the mixed land-use area. Agriculture occupies more space which increases the total rent.

Inside the regulated area, we have:

$$\frac{\partial R_a}{\partial \sigma} \bigg|_{[\hat{x}, \pi]} = 2\pi \int_{\hat{x}}^{x} \frac{\partial \Theta(x)}{\partial \sigma} r_a(x) dx + 2\pi \int_{\hat{x}}^{x_2} \frac{\Theta(x)}{\partial \sigma} \frac{\partial r_a(x)}{\partial \sigma} dx \quad (52)$$

The effect of an increase in $\sigma$ in the regulated area is positive for two reasons: the increase in the proportion of farmland and the increase in bid-rents, due to the subsidy.

In our study case, where the regulated area is fully comprised within the peri-urban area, the overall effect of the AEP on the agricultural part of the total rent is positive.