Newcastle University

MATHEMATICS REVISION BOOKLET

– helping you to succeed in Higher Education –

2nd Edition (c) 2002
School of Mathematics, Statistics and Physics
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Newcastle University
www.ncl.ac.uk/maths-physics

Introduction

What is this booklet for?

This booklet is to help you prepare for your degree programme at University. It includes material on a number of mathematical topics that you have probably met at school or college, and which we shall be using frequently in the modules you will be covering. If you can use these techniques easily and reliably, you will be better able to understand the new topics you meet, and to achieve success in your degree.

If you are not fluent in these techniques, or if there are any you have not met at all, then it is important that you try to prepare yourself before arrival at University. This booklet is designed to help you do so. We hope you find it useful, and that you arrive in Newcastle ready to tackle a challenging and interesting degree with excellent employment prospects!

Note for students without A-level or equivalent in Mathematics (including Engineering Foundation Year)

This booklet is designed to be suitable for those entering with a variety of background qualifications in mathematics. If you have *not* done A-level, then some of the topics covered in this booklet will be taught in your first year at Newcastle, so don’t worry if you haven’t seen them before (especially section 8 on calculus). However, the earlier sections, especially 1, 2, 4 and 5, are extremely important and you should try to complete as much as possible of these parts at least.

How should I use it?

Here are some suggestions about how to use the booklet to best effect:

1. Do your working in the boxes below each question. If you run out of space, use extra paper (this is *not* going to be marked!).
2. Do not use your calculator. The arithmetic is straightforward, and we think you should be able to do it, except for some cases where the answer is left in e.g. the form $\sqrt{13}$.
3. Try at least a few exercises from *every* section, even if you are already confident. This will help you to 'brush up' on material that you have not used for a while.
4. Questions marked * have no answers given. We have given answers to most exercises, but not all. These are there to test your confidence, so check your working carefully for each of them!
5. If you are not really confident on any section, then you should attempt *all* the exercises, and perhaps try to find a textbook (or past notes) to help you reinforce these topics. If you get stuck on any section, try a later one – don't stop there!

Is it assessed?

This material should mostly have been in your previous GSCE, A-level or other qualifications. We will not assess your knowledge of it directly, unless it *is* taught in your normal modules, but much of it will be used as part of your courses.

What happens when I arrive at University?

When you arrive, you may be given an initial 'diagnostic test', either computer-based or pen-and-paper, to help you identify any weaknesses, and to help us find whether you need extra help in the early stages of your degree. Your department or school may then offer extra classes to help you catch up and cope, or the University may offer classes or support. Please bring this booklet with you; you can continue to work on these topics on your own or in class.

For courses in subjects such as Engineering or Physics, it will assumed that you know most of this material already. For other degree courses, some topics will be taught in your normal classes. You will get more information on arrival about this.

Computer-based test and self-testing

The computer-based test *DIAGNOSYS* is used for new Engineering students at Newcastle and at a number of other universities, and covers similar material.

Use of DIAGNOSYS at Newcastle

The software is available on all PC clusters. To run it, do the following:

Start - Programs - Departmental - Maths - Diagnosys 3 Maths

or you may choose the icon for 'Self-test' instead. The self-test allows you to choose topic areas, try a variety of questions, and use hints to help if you are unsure. The full test does not give you choice or hints, but uses similar questions and the same interface.

Acknowledgements

This booklet was written by Dr John Appleby of the Engineering Mathematics group in the School of Mathematics, Statistics and Physics, with contributions from other staff in Engineering schools. It is based on a similar book produced by Dr Tony Croft of Loughborough University, to whom thanks are due.

Contents

1. Numbers etc.

1.1 Round each of the following to the number of decimal places given afterwards in brackets:

a)	3.1415924 (3)	b)	3.1415924 (4)	c)	1.4142 (2)	d)	0.003299 (3)
e)	100.0423 (1)	*f)	0.00409 (2)	g)	–34.567 (1)	*h)	0.5499 (1)

1.2 Round each of the following to the number of significant figures given afterwards in brackets:

a)	0.003299 (2)	b)	100.0463 (5)	c)	100.0463 (4)	d)	1473.3 (2)
e)	14.548 (3)	*f)	–0.5557 (2)	g)	0.0000034 (5)	*h)	17001.3 (1)

1.3 Put each number into scientific (standard) notation, e.g. 3.45×10^{-3}. Also, put into a form using only powers of 10 that are multiples of 3, i.e. 10^3, 10^6, 10^{-3}, 10^0, etc.

a)	34.56	b)	1089.4	c)	0.3027	d)	0.000552
e)	–5.63	*f)	–1001.1001	g)	0.0000004	*h)	–99000.0

1.4 Convert each number to ordinary decimal notation, e.g. 0.000403 :

a)	3.5×10^{-3}	b)	-2.071×10^{5}	c)	9.930×10^{-1}	d)	0.207×10^{-2}
e)	4.2156×10^{4}	*f)	3.14159×10^{2}	g)	1.00×10^{6}	*h)	-4×10^{-7}

1.5 Put a $<$ or $>$ sign between each pair of numbers to show which is greater and which is less:

a)	9901 10032	b)	–3 –7	c)	4 –17	d)	–99.43 0.02
e)	0 –0.341	*f)	10^2 10^5	g)	10^{-2} 10^{-3}	*h)	-4×10^3 3×10^4

1.6 In each case, multiply the number by 100, also divide it by 1000:

a)	3.1416	b)	–453.001	c)	207.1	d)	–0.00302
e)	3×10^4	*f)	7.5×10^2	g)	-3.2×10^{-3}	*h)	27.4×10^{-2}

1.7 Write each decimal as a fraction, and each fraction as a decimal:

a)	0.2	b)	0.125	c)	–0.75	d)	0.375
e)	1/4	*f)	–7/5	g)	3/40	*h)	1/12

1.8 Find the Highest Common Factor and Lowest Common Multiple of each pair or triple of numbers (e.g. 8 and 12 have HCF of 4 and LCM of 24):

a)	4 8	b)	6 9	c)	4 15	d)	6 8
e)	10 15	*f)	12 15	g)	5 12	*h)	30 40 55

1.9 Calculate each of the following:

a) $(-3) \times (4)$, b) $(-5) \times (-7)$, c) $-(-5) \times (-4)$, d) $(-1.5) \times (-4)$, e) $7 - 3 \times 4$, *f) $-5 - 4 \times 2$
g) $4 - \frac{3}{5} - \frac{1}{5}$, *h) $15 \div 6 \div 2$ (what result would your calculator give for this – think before trying it!)

2. Fractions, Indices and Logarithms

2.1 Cancel each fraction to leave it in its simplest form (e.g. $\frac{18}{12}$ becomes $\frac{3}{2}$):

a) $\frac{6}{4}$, b) $\frac{30}{18}$, c) $-\frac{12}{9}$, d) $\frac{36}{28}$

2.2 Simplify each expression, leaving the result as a single improper fraction (e.g. $\frac{11}{8}$ not $1\frac{3}{8}$):

a) $\frac{1}{2} + \frac{1}{3}$ b) $\frac{2}{3} + \frac{5}{6}$ c) $\frac{3}{4} - \frac{5}{6}$ *d) $\frac{1}{3} + \frac{3}{4} - \frac{1}{5}$

2.3 Multiply or divide fractions, simplifying the result in each case:

a) $\frac{2}{3}\times\frac{1}{4}$ b) $\frac{3}{7}\times\frac{49}{6}$ *c) $-\frac{3}{4}\times\frac{6}{5}$ d) $\left(-\frac{9}{5}\right)\times\left(-\frac{15}{8}\right)$

e) $\frac{2}{3}\div\frac{1}{6}$ f) $\frac{3}{4}\div\frac{2}{3}$ g) $-\frac{1}{5}\div\frac{3}{5}$ *h) $\frac{5}{12}\div\frac{10}{9}$

2.4 Put over the lowest common denominator, and simplify the result, in each case (e.g. $\frac{1}{x}-\frac{3}{x^2}=\frac{x-3}{x^2}$):

a) $\frac{3}{x-1}+\frac{2}{x+2}$, b) $\frac{4}{y+3}-\frac{3}{y-3}$, c) $\frac{1}{2x+3}+\frac{4}{x-2}$, d) $\frac{a}{y}-\frac{a+2}{2y-1}$ e) $\frac{3}{u}-\frac{2}{u+1}+\frac{4}{u-2}$,

f) $\frac{1}{2}z+\frac{3}{z}-\frac{2}{z^2}$, g) $1+\frac{2x-3}{x^2+1}-\frac{3}{4x}$, *h) $2x-1+\frac{3}{2(x-2)}-\frac{1}{2(x+3)}$ i) $\frac{2}{x(x-1)^2}-\frac{3}{x^2(x-1)}$

2.5 Simplify each expression:

a) $\left(\frac{x}{2}\right)\left(\frac{3x^2}{4}\right)$, b) $\left(\frac{x^2}{6}\right)\left(\frac{9}{x}\right)$, c) $\left(\frac{2x^2y}{5z}\right)\left(\frac{3yz^2}{4x^4}\right)$, d), $\left(\frac{1}{x^2}\right)\div\left(\frac{2}{x}\right)$, e), $\left(\frac{a}{a+1}\right)\div\left(\frac{3a^2}{a^2+1}\right)$, *f) $\left(\frac{x}{2yz}\right)\div\left(\frac{x^2}{y(z+1)}\right)$, g) $\left(\frac{3b}{b+1}\right)\div\left(\frac{2b}{b^2-1}\right)$, *h) $\left(\frac{3x+6}{2y^3}\right)\div\left(\frac{6x}{3yz}\right)$

2.6 Give the value of each of the following:

a) 2^{-3}, b) 13^0, *c) 4^{-1}, d) $(½)^{-2}$, e) $8^{-\frac{1}{3}}$, f) $-16^{\frac{1}{4}}$, g) $27^{-\frac{2}{3}}$, *h), $32^{\frac{3}{5}}$

2.7 Simplify each of the following (e.g. $10^2 \times 10^3 = 10^5$):

a) $10^3 \times 10^4$ b) $2^5 \times 2^{-3}$ c) $x^{0.5} x^{1.5}$ d) x^5 / x^2

e) $5^7 \div 5^3$ f) $y^{-\frac{1}{2}} y^{-\frac{5}{2}}$ g) $(2x)^3$ h) $\left(4x^2\right)^{\frac{1}{2}}$

i) $\left(x^2\right)^3$ *j) $\left(3y^{\frac{3}{2}}\right)^2$ k) $\left(z^{-1}\right)^3$ *l) $\left(x^{-1}\right)^{-1}$

m) $\left(9x^{-2}\right)^{\frac{1}{2}}$ *n) $\left(10^{-3.5}\right)^0$ o) $\left(\frac{1}{2} w^3\right)^{-1}$ *p) $\left(3u^2\right)^{-1}\left(2u^{-3}\right)^4$

2.8 Give the value of each:

a) $\log_{10}(100)$ b) $\log_{10}(1000000)$ c) $\log_{10}(0.1)$ d) $\log_{10}\left(\frac{1}{1000}\right)$

e) $\log_2(8)$ *f) $\log_2\left(\frac{1}{16}\right)$ g) $\log_e\left(e^{-3}\right)$ *h) $\log_e\left(\frac{1}{e}\right)$

2.9 Give the result as the logarithm of a single number, or as a value where possible:

a) $\log 3 + \log 4$ b) $\log 16 - \log 2$ c) $3\log 2 - 2\log 4$ d) $-\log \frac{1}{2}$

e) $10^{\log_{10} 5}$ *f) $-2e^{\ln 2}$ g) $\log_e 1$ *h) $e^{-\ln x}$

3. Units and dimensions

Standard prefixes: m = milli = 10^{-3}, k = kilo = 10^{3}, M = mega = 10^{6}, μ = micro = 10^{-6}, (also c = centi = 10^{-2}).

SI Units: m = metres, s = seconds, N = newtons, kg = kilogrammes, V = volts, F = farads, C = coulombs, J = joules, (also l = litres).

For this exercise, use $g \approx 10ms^{-2}$ (more accurate value: $9.81ms^{-2}$).

3.1 How many ...... ? (e.g. How many mm in one m? Answer: 10^3)

a) cm in one m b) ml in $2.5l$ c) s in $1hr\ 30mins$ d) mV in $2kV$

e) m^2 in $5km^2$ *f) μF in $2.3F$ g) μm in $50.3mm$ *h) degrees in 2.5 revolutions

3.2 Convert to the units given:

a) $5m^2$ to mm^2,   b) $3ms^{-1}$ to km/hr,   c) $5Nmm^{-2}$ to Nm^{-2},   *d) $g.mm^{-2}$ to $kg.m^{-2}$

3.3 Give the value in each case (with units):

a) $3.6m$ / $3mm$,   b) $10m^2$ / $2.5m$,   c) $4ms^{-1} \times 5mins$,   *d) ρgh, where $\rho = 3kg.m^{-3}$, $h = 2m$

3.4 A (straight-line) graph of pressure (y axis) against 1/Volume (x) goes through the points ($1/V = 5m^{-3}$, $p = 2000$ Nm^{-2}), ($1/V = 15m^{-3}$, $p = 4000$ Nm^{-2}). Find the gradient (with units).

3.5 If the area of a rectangle is $3.6m^2$, and one side has length $180mm$, what is the length of the other side in m?

3.6 The Reynold's number Re for a fluid flow is given by $\mathrm{Re} = \dfrac{\rho U L}{\mu}$ (the quantities are density, velocity, length and viscosity).
If $\rho = 1000 kg.m^{-3}$, $U = 3.5 m.s^{-1}$, $L = 0.2 m$, $\mu = 0.01 kg.m^{-1}.s^{-1}$, calculate Re and its units.

***3.7** Water has density $1 g / cm^3$. If steel has density 7.8 times that of water, what is the density of steel in tonnes per cubic metre (1 t = 1000 kg)?

4. Brackets, factors, simplification

4.1 Expand and collect terms:

a) $2x(3-4x)$, b) $u-1+2(3-u)$, c) $2y+4(y-2)$, d) $z(z+2)-(z-2)$,
e) $3u(u+2)-2u(4-u)$, *f) $5x(2x-3)-1+2x-x(4-3x)$

4.2 Expand and collect terms:

a) $(x+1)(x+4)$, b) $(u-3)(u+4)$, c) $(y+3)(y-3)$, *d) $(2z-1)(2z+1)$,
e) $(x+2)(x+y-3)$, f) $(2x-y+3)(3x+2y-2)$, g) $(x+y)^2-(x-y)^2$,
h) $(a+b)^3$, *i) $(x+1)(x^2-x+1)$, j) $\left(x+\frac{1}{x}\right)^2$, k) $\left(\frac{1}{x}-\frac{2}{x^2}\right)^2$

4.3 Factorise as far as possible in each case (e.g. $18x^2y+8xy^2=2xy(9x+4y)$):

a) $6x+3xy$, b) $8u^2-6uv$, c) $\frac{1}{2}x^3-\frac{1}{4}x^2$, *d) $4ab^2c-9ac^3$, e) $3x^2-12xyz+15xz^2$,
f) $\frac{u}{4v^2}-\frac{u^2}{6v}$, g) $\sqrt{x}+x$, h) $4\sqrt{1-2x}-6(1-2x)$

4.4 Factorise into two brackets:

a) x^2+3x+2, b) y^2-4y+3, c) $v^2-2v-15$, *d) z^2+z-20, e) $2x^2+3x+1$,
*f) $2u^2+u-3$, g) $6z^2-13z+6$, h) x^2+6x+9, i) a^2-b^2, j) $4x^2-9y^2$, k) x^2-3,
l) u^2-4u+4

4.5 Factorise into three brackets, where one is given:

a) $x^3-3x^2+3x-1=(x-1)(.......)$, b) $y^3-5y^2+2y+8=(y+1)(........)$

4.6 Given $f(x)$ in each case, write down and simplify $f(2x-1)$ and $f(1/y)$:

a) $f(x)=x^2-3x$, b) $f(x)=1-2x+3x^2$, c) $f(x)=\dfrac{x}{2}+\dfrac{2}{x}$, *d) $f(x)=(1-x^2)^2$

4.7 Write each of the following in the form $(x+a)^2+b$,
(e.g. $x^2+2x+2=[(x+1)^2-1]+2=(x+1)^2+1$) – 'completing the square'.

a) x^2+4x+5, b) x^2-6x+8, c) x^2+3x+3, d) x^2-x+1, *e) $5-7x+x^2$

5. Formulae and equations

5.1 Solve each equation:

a) $2x-3=4-3x$, b) $4y+1=\frac{1}{2}y-\frac{3}{2}$, c) $\dfrac{3-z}{2}=\dfrac{1-3z}{4}$, d) $\dfrac{1}{u}=\dfrac{3}{u-1}$,
e) $\dfrac{2x-1}{3x+1}=\dfrac{4-2x}{1-3x}$, * f) $\dfrac{z+1}{z-3}=\dfrac{4-z}{2-z}$

5.2 Transpose each formula to make the bracketed variable the subject:

a) $x = 3t - 2\ \ (t)$, b) $y = \dfrac{3}{2-x}\ \ (x)$, c) $T = 2\pi\sqrt{\dfrac{L}{g}}\ \ (L)$, d) $u = \dfrac{3v-2}{2v+1}\ \ (v)$,

e) $Q = \dfrac{\pi a^4 p}{8\mu L}\ \ (p)$, f) $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}\ \ (R)$, *g) $pV^{\gamma} = C\ \ (\gamma)$

5.3 Solve each quadratic equation *by factorising*:

a) $x^2 - 3x + 2 = 0$, b) $y^2 - y - 12 = 0$, *c) $z^2 + 2z - 24 = 0$, d) $2u^2 - u - 1 = 0$,
e) $v - 3v^2 + 2 = 0$, f) $4w^2 + 3w - 10 = 0$, g) $x^2 = 4x$, *h) $-3y = -5y^2$

5.4 Solve each quadratic equation *by completing the square* (see Exercise 4.7):

a) $x^2 + 4x + 3 = 0$,   b) $y^2 - 2y = 0$,   c) $z^2 + z - 2 = 0$,   d) $3 - 4u + u^2 = 0$,

e) $1 - v^2 - 4v = 0$,   f) $2w^2 + 2w - 1 = 0$,   g) $ax^2 + bx + c = 0$ (a, b, c constant)

(Hint: for e), f), g) take out a factor to make the coefficient of the square equal to 1.)

5.5 The solution of question 5.4g) gave the standard formula for solving quadratic equations. Use it to solve again the quadratic equations in 5.4a) to f).

5.6 Given the values of the roots, write down a quadratic or cubic equation with those roots (e.g. given 2,3, obtain $x^2 - 5x + 6 = 0$):

a) 3, 4,   b) 2, -1,   c) ½, -2,   d) 1, 2, 3,   e) -1, 3, -4

5.7 Solve the simultaneous equations:

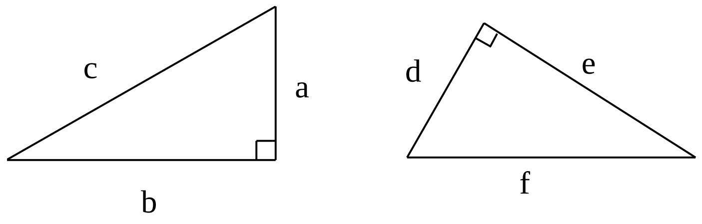
a) $\begin{matrix} 2x-3y=4 \\ -x+2y=5 \end{matrix}$, b) $\begin{matrix} 4x+3y=0 \\ 3x-5y=2 \end{matrix}$, c) $\begin{matrix} 2u+v=3 \\ \frac{1}{2}u+\frac{3}{2}v=1 \end{matrix}$, *d) $\begin{matrix} x^2-y=0 \\ 2x+y=-1 \end{matrix}$

5.8 For each, find a pair of values (choose small integers) for *x*,*y* that satisfy the equation:

a) $x-y=0$, b) $x+y=0$, c) $2x+y=0$, d) $3x-2y=0$, *e) $\frac{1}{2}x+2\mathrm{y}=0$, f) $-4x-3y=0$

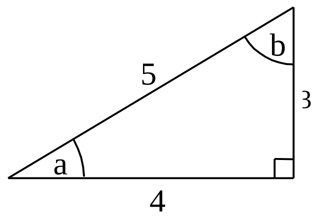
6. Trigonometry etc.

6.1 In each case, given two sides, find the third side (your answer may be left as a square root if you can't calculate it exactly, e.g. √7).



a) $a = 3$, $b = 5$, find c, b) $a = 4$, $c = 7$, find b, c) $d = 5$, $e = 12$, find f, d) $e = 6$, $f = 7$, find d.

6.2 For the triangle shown, write down $\sin a$, $\cos a$, $\tan a$, $\sin b$, $\operatorname{cosec} a$, $\cot b$, $\sec a$



6.3 Give the values of $\sin 30°,\ \sin 45°,\ \sin 60°,\ \cos 0°,\ \tan 45°,\ \cos 30°,\ \sin 90°,\ \sin 0°,\ \cos 90°$ *without using a calculator*, and find the value of $(\sin\theta)^2 + (\cos\theta)^2$ for several values of θ.

6.4 Convert between degrees and radians:

a) 30º, b) 90º, *c) 45º, d) 150º, e) 270º, f) 315º, g) π/2, *h) π/4, i) 2π/3, j) 5π/4, *k) 9π/4

6.5 Give values of

$\tan 60°, \sin 120°, \sin(3\pi/4), \sin 210°, \cos(5\pi/6), \tan 225°, \cos(3\pi/2), \sin(-30°)$

6.6 Convert between polar and cartesian coordinates in each case:

a) $x = 4$, $y = 3$, b) $x = -1$, $y = 1$, c) $x = 1$, $y = -\sqrt{3}$, d) $r = 3$, $\theta = 45^{\circ}$, e) $r = 2$, $\theta = 120^{\circ}$, f) $r = 4$, $\theta = 150^{\circ}$, *g) $r = 2\sqrt{2}$, $\theta = 135^{\circ}$

6.7 Find the equation of a circle from the information provided:

a) Centre at (0,0), radius 3, b) Centre at (2,-3), radius 4.

7. Graphs

7.1 Find the equation of the straight line in each case (use variables *x*,*y*):

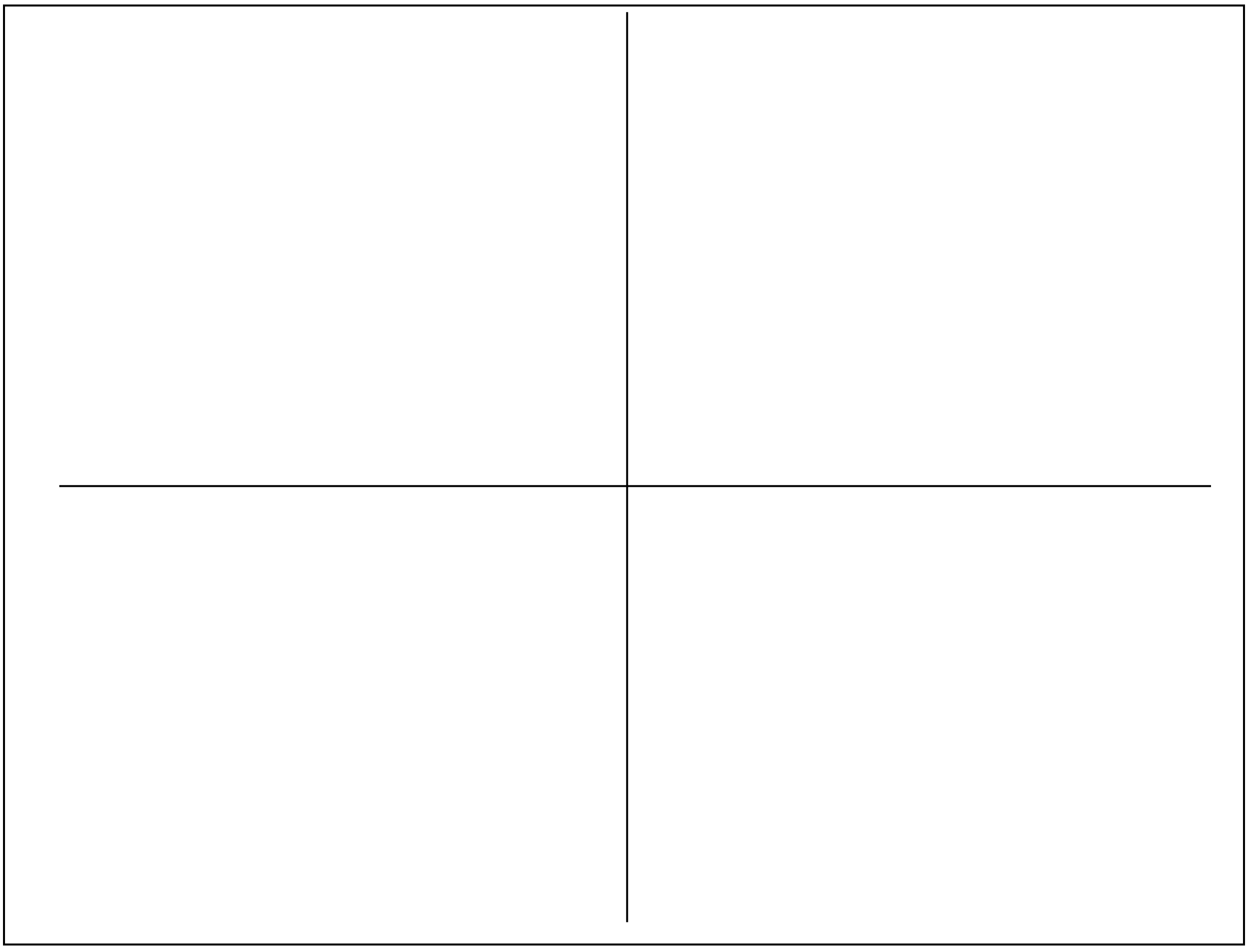
a) through (0,0) and (3,6), b) through (-1,3) and (1,1), c) through (-3,-4) and (-5,-1),
d) through (2,-3) with gradient ½, e) through (1,3), perpendicular to the line $y = 1 - 2x$

7.2 Sketch the graphs (all on one diagram) of:

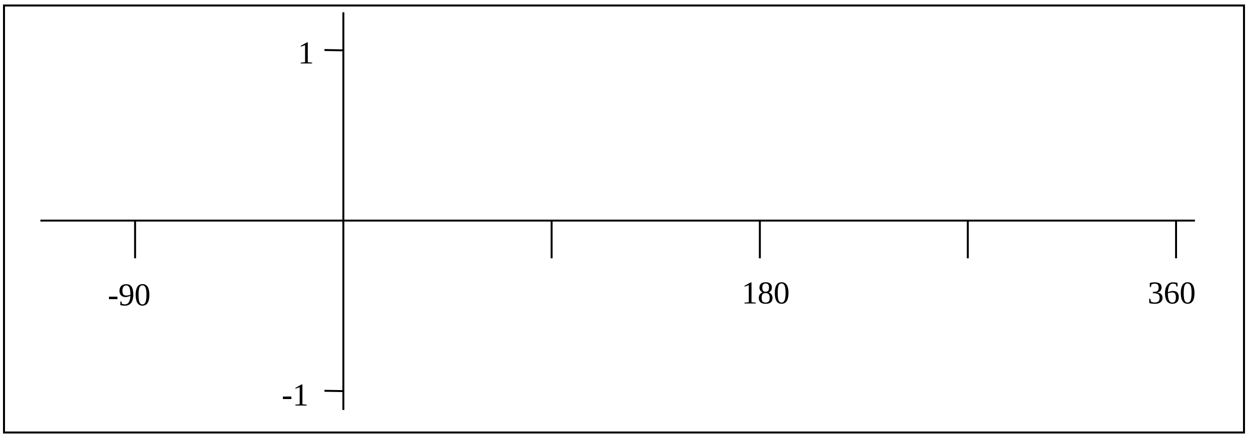
a) $y = 2x - 1$, b) $2x + 4y - 1 = 0$, c) $x = 3$, d) $y = -2$, and give the gradient for case b).

7.3 Sketch (on one diagram) the graphs of

a) $y = x^2 - 1$, b) $y = 4 - x^2$, c) $y = -\frac{1}{x}$, d) $y = 2 - \frac{3}{x}$, e) $y^2 = 4x$



7.4 Sketch the graphs of $\sin x$, $\cos x$, $\sin 2x$ for values of x from -90º to 360º.



8. Differentiation and simple integration

8.1 Differentiate each expression (note a is a constant):

a) $y = x^2 - 1$, b) $y = \tfrac{1}{2}x^3 - 3 + 2ax$, c) $z = -\dfrac{1}{u}$, d) $v = \dfrac{u - 3u^2}{4a}$, e) $z = \dfrac{4}{3u^2} - \dfrac{\sqrt{u}}{5}$

8.2 Differentiate each expression (a is constant): a) $y = \cos x$, b) $v = e^{2u}$, c) $f = a\sin 3\theta$,

d) $s = 2\ln t - 3e^{-t} - 3a$, e) $y = 2a\pi \tan x - \dfrac{a-1}{x}$

8.3 Differentiate:

a) $y = 3x^2 \sin x$, b) $z = \dfrac{2-3w}{1+4w}$, c) $s = \cos(t^2)$, d) $w = \dfrac{3m - 2\sqrt{m}}{m^3}$

8.4 Give the indefinite integral of:

a) $x^3 - 3x + 1$, b) $-\cos x + e^{-x} + \dfrac{3}{x}$, c) $4\sin 2x - \frac{1}{2}\sec^2 x$, d) $\dfrac{t\sqrt{t} - 3}{t^2}$

Answers to Exercises

Exercises marked (*) have *no* answers given here. You must check these carefully yourself!

Also note that many answers have *alternative forms*. For example, whether or not you expand brackets, factorise or put fractions over a common denominator is often a matter of choice. So, if your answer is in a slightly different form, but which is equivalent, that's acceptable.

1.1 a) 3.142, b) 3.1416, c) 1.41, d) 0.003, e) 100.0, f) *, g) -34.6, h) *

1.2 a) 0.0033, b) 100.05, c) 100.0, d) 1500, e) 14.5, f) *, g) 0.0000034000, h) *

1.3 a) 3.456×10^1 or 34.56×10^0, b) 1.0894×10^3, c) 3.027×10^{-1} or 0.3027×10^0, d) 5.52×10^{-4} or 0.552×10^{-3}, e) -5.63×10^0, f) *, g) 4×10^{-7}, or 0.4×10^{-6}, h) *

1.4 a) 0.0035, b) -207100, c) 0.9930, d) 0.00207, e) 42156, f) *, g) 1000000, h) *

1.5 a) $<$, b) $>$, c) $>$, d) $<$, e) $>$, f) * , g) $>$, h) *

1.6 a) 314.16, 0.0031416 or 3.1416×10^{-3}, b) -45300.1, -0.453001, c) 20710, 0.2071, d) -0.302, -0.00000302, e) 3×10^6, 30, f) *, g) -3.2×10^{-1}, -3.2×10^{-6}, h) *

1.7 a) 1/5, b) 1/8, c) -3/4, d) 3/8, e) 0.25, f) * , g) 0.075, h) *

1.8 a) 4, 8, b) 3, 18, c) 1, 60, d) 2, 24, e) 5, 30, f) * , g) 1, 60, h) *

1.9 a) -12, b) 35, c) -20, d) 6, e) -5, f) * , g) 16/5, h) *

2.1 a) 3/2, b) 5/3, c) -4/3, d) 9/7   2.2 a) 5/6, b) 3/2, c) -1/12, d) *

2.3 a) 1/6, b) 7/2, c) * , d) 27/8, e) 4, f) 9/8, g) -1/3, h) *

2.4 a) $\dfrac{5x+4}{x^2+x-2}$, b) $\dfrac{y-21}{y^2-9}$, c) $\dfrac{9x+10}{2x^2-x-6}$, d) $\dfrac{ay-2y-a}{2y^2-y}$,
e) $\dfrac{5u^2+5u-6}{u^3-u^2-2u}$, f) $\dfrac{z^3+6z-4}{2z^2}$, g) $\dfrac{4x^3+5x^2-8x-3}{4x^3+4x}$, h) *
i) $\dfrac{3-x}{x^2(x-1)^2}$

2.5 a) $\dfrac{3x^3}{8}$, b) $\dfrac{3x}{2}$, c) $\dfrac{3y^2z}{10x^2}$, d) $\dfrac{1}{2x}$, e) $\dfrac{a^2+1}{3a(a+1)}$, f) * , g) $\dfrac{3(b-1)}{2}$, h) *

2.6 a) 1/8, b) 1, c) * , d) 4, e) 1/2, f) -2, g) 1/9, h) *

2.7 a) 10^7, b) 2^2, c) x^2, d) x^3, e) 5^4, f) y^{-3}, g) $8x^3$, h) $2x$, i) x^6, j) * , k) z^{-3}, l) * , m) $3x^{-1}$, n) * , o) $2w^{-3}$, p) *

2.8 a) 2, b) 6, c) -1, d) -3, e) 3, f) * , g) -3, h) *

2.9 a) log 12, b) log 8, c) log ½ , d) log 2, e) 5, f) * , g) 0, h) *

3.1 a) 100, b) 2500, c) 5400, d) 2×10^6, e) 5×10^6, f) * , g) 50300, h) *

3.2 a) 5×10^6 mm^2, b) 10.8 km/hr, c) 5×10^6N.m^{-2}, d) *

3.3 a) 1200, b) 4 m, c) 1200 m, d) *   3.4 200 N.m   3.5 20 m

3.6 Re = 7×10^4 (*no* units, or 'dimensionless', which is why it's called Reynold's *number*)

3.7 *

4.1 a) $6x-8x^2$, b) $5-u$, c) $6y-8$, d) z^2+z+2, e) $5u^2-2u$, f) *

4.2 a) x^2+5x+4, b) u^2+u-12, c) y^2-9, d) *, e) $x^2+xy+2y-x-6$, f) $6x^2+xy-2y^2+5x+8y-6$, g) $4xy$, h) $a^3+3a^2b+3ab^2+b^3$, i) *, j) $x^2+2+\dfrac{1}{x^2}$, k) $\dfrac{1}{x^2}-\dfrac{4}{x^3}+\dfrac{4}{x^4}$

4.3 a) $3x(2+y)$, b) $2u(4u-3v)$, c) $\frac{1}{4}x^2(2x-1)$, d) *, e) $3x(x-4yz+5z^2)$, f) $\dfrac{u}{2v}\left(\dfrac{1}{2v}-\dfrac{u}{3}\right)$ or $\dfrac{u}{12v^2}(3-2uv)$, g) $\sqrt{x}(1+\sqrt{x})$, h) $2\sqrt{1-2x}\left(2-3\sqrt{1-2x}\right)$

4.4 a) $(x+2)(x+1)$, b) $(y-3)(y-1)$, c) $(v-5)(v+3)$, d) *, e) $(2x+1)(x+1)$, f) *, g) $(3z-2)(2z-3)$, h) $(x+3)^2$, i) $(a+b)(a-b)$, j) $(2x+3y)(2x-3y)$, k) $\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)$, l) $(u-2)^2$

4.5 a) $(x-1)^3$, b) $(y+1)(y-2)(y-4)$

4.6 a) $4x^2-10x+4$ or $2(2x^2-5x+2)$, $\dfrac{1-3y}{y^2}$, b) $12x^2-16x+6$, $\dfrac{y^2-2y+3}{y^2}$, c) $\dfrac{4x^2-4x+5}{4x-2}$, $\dfrac{1+4y^2}{2y}$ or $2y+\dfrac{1}{2y}$, d) *

4.7 a) $(x+2)^2+1$, b) $(x-3)^2-1$, c) $\left(x+\frac{3}{2}\right)^2+\frac{3}{4}$, d) $\left(x-\frac{1}{2}\right)^2+\frac{3}{4}$, e) *

5.1 a) x = 7/5, b) y = -5/7, c) z = -5, d) u = -1/2, e) x = -1, f) *

5.2 a) $t=\dfrac{x+2}{3}$, b) $x=2-\dfrac{3}{y}$, c) $L=\dfrac{gT^2}{4\pi^2}$, d) $v=\dfrac{2+u}{3-2u}$, e) $p=\dfrac{8Q\mu L}{\pi a^4}$, f) $R=\dfrac{R_1R_2}{R_1+R_2}$, g) * (use logs)

5.3 a) $(x-2)(x-1)=0$ so x = 1 or 2, b) y = 4, -3, c) *, d) u = 1, -1/2, e) v = 1, -2/3, f) w = 5/4, -2, g) x = 0, 4, h) *

5.4 a) $(x+2)^2-1=0$, $x+2=\pm 1$, $x=-1,-3$, b) y = 0, 2, c) z = 1, -2, d) u = 1, 3, e) $v=-2\pm\sqrt{5}$, f) $w=\dfrac{-1\pm\sqrt{3}}{2}$, g) $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ (after simplifying)

5.5 – as for 5.4, but using the formula instead of completing the square

5.6 a) $(x-3)(x-4)=0$, $x^2-7x+12=0$, b) $x^2-x-2=0$, c) $2x^2+3x-2=0$, d) $x^3-6x^2+11x-6=0$, e) $x^3+2x^2-11x-12=0$

5.7 a) x = 23, y =14, b) (6/29, -8/29), c) (7/5, 1/5), d) *

5.8 a) (1, 1) or any with same ratio, b) (1, -1), c) (1, -2), d) (2, 3), e) *, f) (3, -4)

6.1 a) $c=\sqrt{34}$, b) $b=\sqrt{33}$, c) f = 13, d) $\sqrt{13}$

6.2 sin a = 3/5, cos a = 4/5, tan a = 3/4, sin b = 4/5, cosec a = 5/3, cot b = 3/4, sec a = 5/4

6.3 $\frac{1}{2},\ \frac{1}{\sqrt{2}},\ \frac{\sqrt{3}}{2},\ 1,\ 1,\ \frac{\sqrt{3}}{2},\ 1,\ 0,\ 0$. Also, note that $(\sin\theta)^2+(\cos\theta)^2=1$ for *all* values of θ.

6.4 a) π/6, b) π/2, c) * , d) 5π/6, e) 3π/2, f) 7π/4, g) 90°, h) * , i) 120°, j) 225°, k) *

6.5 $\sqrt{3},\ \frac{\sqrt{3}}{2},\ \frac{1}{\sqrt{2}},\ -\frac{1}{2},\ -\frac{\sqrt{3}}{2},\ 1,\ 0,\ -\frac{1}{2}$

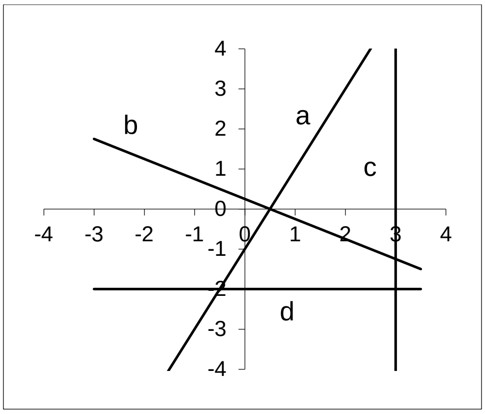
6.6 a) $r=5,\ \theta=\tan^{-1}\left(\frac{3}{4}\right)$, b) $r=\sqrt{2},\ \theta=135°$, c) $r=2,\ \theta=-60°$,

d) $x=\frac{3}{\sqrt{2}},\ y=\frac{3}{\sqrt{2}}$, e) $x=-1, y=\sqrt{3}$, f) $x=-2\sqrt{3},\ y=2$, g) *

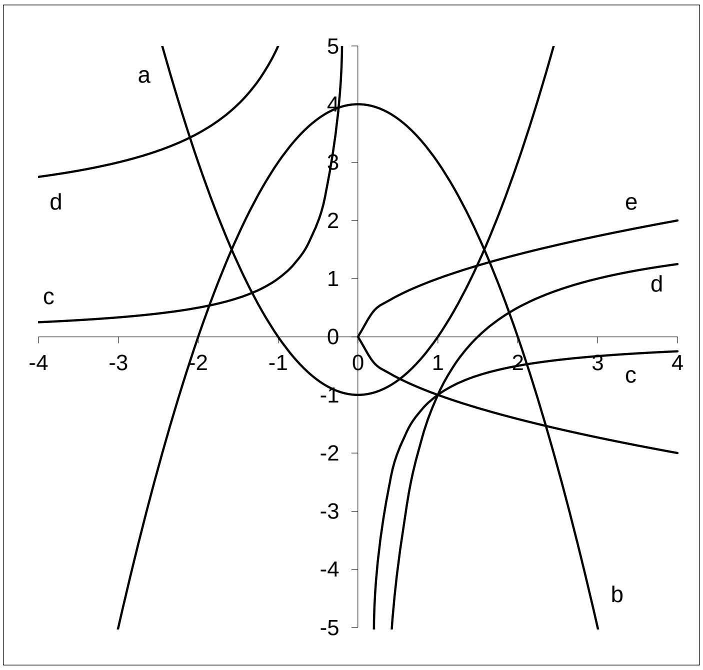
6.7 a) $x^2+y^2=9$, b) $(x-2)^2+(y+3)^2=16$ or $x^2+y^2-4x+6y-3=0$

7.1 a) $y=2x$, b) $y=2-x$, c) $y=-\frac{3}{2}x-\frac{17}{2}$, d) $y=\frac{1}{2}x-4$, e) $y=\frac{1}{2}x+\frac{5}{2}$

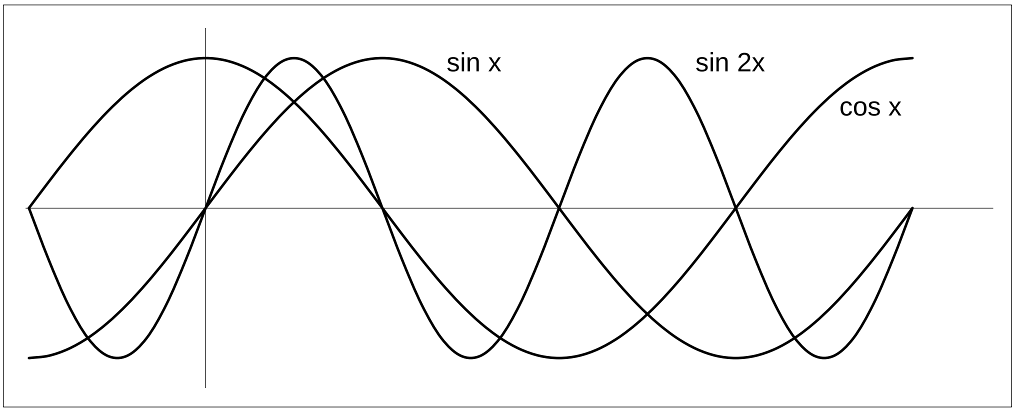
7.2 Case b), gradient is - 0.5



7.3



7.4



8.1 a) $\frac{dy}{dx} = 2x$, b) $\frac{dy}{dx} = \frac{3}{2}x^2 + 2a$, c) $\frac{dz}{du} = \frac{1}{u^2}$, d) $\frac{dv}{du} = \frac{1-6u}{4a}$,

e) $\frac{dz}{du} = -\frac{8}{3u^3} - \frac{1}{10\sqrt{u}}$

8.2 a) $\frac{dy}{dx} = -\sin x$, b) $\frac{dv}{du} = 2e^{2u}$, c) $\frac{df}{d\theta} = 3a\cos 3\theta$, d) $\frac{ds}{dt} = \frac{2}{t} + 3e^{-t}$,

e) $\frac{dy}{dx} = 2a\pi \sec^2 x + \frac{a-1}{x^2}$

8.3 a) $\frac{dy}{dx} = 6x\sin x + 3x^2\cos x$, b) $\frac{dz}{dw} = \frac{-11}{(1+4w)^2}$, c) $\frac{ds}{dt} = -2t\sin\left(t^2\right)$,

d) $\frac{dw}{dm} = -\frac{6}{m^3} + \frac{5}{m^{7/2}} = -\frac{6}{m^3} + \frac{5}{m^3\sqrt{m}}$

8.4 a) $\frac{1}{4}x^4 - \frac{3}{2}x^2 + x + C$, b) $-\sin x - e^{-x} + 3\ln x + C$, c) $-2\cos 2x - \frac{1}{2}\tan x + C$,

d) $2\sqrt{t} + \frac{3}{t} + C$ (NB The constant C is an essential part of the answer!)