Algebra of a Credit Cruch[¤]

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Abstract

This paper develops a simple dynamic version of the Monti-Klein model, based on the empirical evidence on the demand for money. The results suggest that the bank smoothes interest rates shocks, while it does not provide insurance against negative shocks of real origin. The bank, in fact, reacts to these last with a credit crunch, because the optimal size of the portfolio depends on the level of expected marginal default costs. Finally, very low level of market interest rates push the bank to substitute deposits with market sources of finance.

1 Introduction

I propose a dynamic version of the Monti-Klein model, where a dynamic problem arises assuming a very simple cost structure. The dynamic properties depend on a single non-linearity: a convex default cost function that captures a fundamental aspect of banking activity, the ability of banks to finance opaque investment projects whose risk other agents cannot price. The structure of the model is based on the empirical evidence that the transaction component of the demand for money is persistent, and therefore the demand for depository services displays a similar degree of persistence. I further introduce a link between the deposits and loans of the individual bank by assuming that the demand for deposit services is affected by the equilibrium quantity of loans. As discussed by Diamond and Rajan (2001a,b) or Kashyap et al. (2002), banks create liquidity issuing liquid deposits backed by illiquid loans. Loans feedback into deposits, because the resources that banks provide by lending increase the deposits of the banking system.[1] A bank benefiting of a monopoly in the market for deposits could choose the size of its liabilities just by managing the assets, as central banks do, thanks to their monopoly in the market for currency. Accordingly, new loans generate deposits in proportion to the market share in the market for deposits. In the case of small banks this effect is strengthened by compensating balance requirements that remain still a relevant feature of this part of the industry. This assumption is supported by Ramey (1992) findings that M1 and the component of M1 consisting of firms' demand deposits are cointegrated, and therefore share the same stochastic trend. In this framework loans become an 'investment' that generates deposits. Default costs are assumed to be a non linear function of the amount of loans issued. As a result, default costs implicitly generate adjustment costs on the stock of deposits, and represent the main constraint on the size of the portfolio of the bank. The equilibrium composition of the portfolio and the size of the portfolio are thus jointly determined.[2] But despite this joint determination of the two, I obtain a very simple mathematical solution,

[1] Whenever a concept such as *inside money* or *endogenous money creation* serves, macroeconomic monetary models implicitly assume this process.

[2] Elsasyani et al. (1995) have not found empirical support for the hypothesis of portfolio separation.

which facilitates analysis of the equilibria.

The main result of the model has a straightforward intuition. When shocks of real nature permanently increase expected default costs, the bank reacts with a credit crunch. The bank, in fact, chooses the optimal amount of loans partially in a backward-looking way, partially forward-looking. The size of the forward looking part depends on expected marginal default costs, so that if these last double, new lending halves. Overall this would suggest that banks amplify the impact of real shocks, generating a financial accelerator, as suggested by Bernanke and Gertler (1991) and Bernanke et al. (1989): when shocks of real origin are expected to raise default costs, for example because the net worth of borrowers decline, the bank reacts by reducing lending and increasing the rate on loans. This story is not at odds with the empirical evidence on the credit crunches of the 1990s in the USA by Bernanke and Lown (1991), Berger and Udell (1994), Brinkmann and Horwitz (1995), and Japan by Woo (2003). These studies highlight that demand side factors, and voluntary risk retrenchment, are at least as important as changes in capital regulation in explaining the credit crunch.

The model also provides some intuition for the empirical results of Gertler and Gilchrist (1993b,a) and Romer and Romer (1990) that commercial and industrial loans issuance increases following tighter monetary policy, for large firms in particular. The model, in fact suggests that following a restrictive monetary policy aimed at raising long-term interest rates, the bank has an incentive to increase the issuance of loans. The bank thus smoothes interest rate shocks. A further implication of the model is that the desired optimal level of deposit services on the part of the banking system is an increasing function of interest rates on bonds.[3] In fact, the optimal supply of deposit services depends on the return of the bond assets.

Finally, the result of the model imply that the bank has a strong incentive to finance direct lending by means of market sources, instead of deposits, when interest rates are

[3]I assume that banks supply deposit services (instead of assuming that they demand deposits) to be consistent with the literature on money demand, where households and firms are assumed to demand deposit services as part of their optimal demand for monetary assets.

particularly low. As a consequence, a long period of very low interest rates may cause an increase of systemic liquidity risk.

This is not the first attempt to describe the banking industry by means of a dynamic model. Cosimano (1987, 1988) analyze how banking intermediation is influenced by monetary policy, in an environment where uncertainty plays a relevant role, because banks must incur adjustment costs to change the stock of loans. Elsasyani et al. (1995) have developed a similar model, where portfolio separation does not hold if banks face adjustment costs for both the stock of loans and the stock of deposits; they provide an empirical estimation of the model, which supports their results. Chami and Cosimano (2001) introduce monopolistic power in this class of models. The main difference among these models regards the specification of the cost functions, which include both convex costs for loans, deposits or funds borrowed from the discount window, and adjustment costs for some of the stocks. The main limitation of these models is that the structure of the results is often quite complicated and the results are not always robust to different specifications of the cost functions. The main innovation of this work is the suggestion that a very simple cost structure can produce a rich enough dynamic framework.

2 The environment

2.0.1 The budget constraint

The banking firm maximizes profits over an infinite horizon, by jointly providing payment services and financial intermediation services. The bank borrows by means of deposits and lends by means of loans; in addition, the bank can either invest the excess of deposits over loans, or borrow some extra resources, by making use of the interbank market. Short-term bonds are a close substitute for interbank lending, since in normal conditions the interbank rate is not too far from the target of the central bank. By further assuming that the bank borrows or lends in the interbank market at the same rate, I can thus focus on the net position of the representative bank by considering its portfolio of bonds (positive or negative).[4] I define as F_t the net interbank position, so that a positive value implies that the bank is a net lender in the market, or alternatively the banks holds bonds in its portfolio of assets instead of issuing them. L_t is the amount of loans, while D_t represents the total amount of all different types of deposits liabilities. Defining the variables in real terms, as ratios with respect to the price level, the budget constraint is the following:[5]

$$L_t + F_t + R_t = D_t + NW, \tag{1}$$

where R_t are the resources held as reserves, and NW is the net worth, initially assumed to be constant. The bank can buy securities or invest in loans only the part of deposits that it does not keep as reserve. Defining with q the legal reserve coefficient, $R_t = qD_t$.[6] Reserves are held in currency or other non interest bearing liabilities of the central bank.[7]

2.0.2 Revenues and costs from payment services

The cost of check clearing and other desk operations is linked to the number of transactions made by the customers and these costs can be formalized as proportional to the

[4]This assumption can be relaxed. The model becomes more complicated, but not substantially different; however it is not very unrealistic for large banks that have implicitly benefited of a state guarantee.

[5]All variables are defined in real terms, which contrasts with the standard practice of the literature on banking, even if it is more in line with the standard assumptions of the theory of the firm. In general, the treatment of the financial sector in real terms suggests the neutrality of money, and it poses the question of price level determinacy. However, I finesse both issues here by introducing the price level as an exogenous process. This permits moving directly from real to nominal variables. Therefore this analysis of the banking firm does not imply money neutrality (since the exogenous price level process can affect real bank behaviour) and does not imply price level indeterminacy. However, it does have an important advantage. Defining the portfolio in real terms makes it possible to have real variables that have stable growth ratios even when the nominal variables diverge. This approach is particularly valuable since the evolution of the financial systems of the last decades has seen a continuous growth of the size of banking intermediaries while the banking industry's share of the financial intermediation has declined.

[6]The bank does not voluntarily hold other free reserves to exclude from the analysis the uncertainty of the demand for deposit services (of deposits supply). This simplification could be relaxed, introducing a penalty cost for the bank's illiquidity, and some non linear adjustment cost for the stock of bonds, to model bonds as imperfectly liquid assets. This complication though, would not generate substantially new results. The model would somewhat replicate the findings of Kashyap et al. (2002), that highlight that the need to hold reserves may be lower when the demand for deposits and the demand for loans are positively correlated.

[7]The model can easily be extended to the case where reserves earn a return. But the results would not not change notably.

average amount of deposits and loans.[8] These costs might be concave, because of an element of fixed costs, but within the context of an infinite horizon problem without entry or exit, fixed costs can be neglected. Besides, the large empirical literature regarding the existence of scale economies in the banking system has not led to undisputed conclusions.[9]

Banks earn fees from transaction services. Assuming that the average number of transaction during a certain period of time is constant, these revenues are proportional to the amount of deposits. Formally:

$$C(D_t) = uD_t \qquad C(L_t) = zL_t, \qquad\qquad R(D_t) = fD_t, \tag{2}$$

where $C(\cdot)$ and $R(\cdot)$ are cost and revenue functions, u, z and f are positive real numbers, L_t represents the amount of loans issued by the bank, and D_t are deposits.

2.0.3 Revenues and costs from financial intermediation

Revenues form financial intermediation services stem from the difference between the interest rate r_{Lt}, that the bank charges on loans, and the interest rate r_{Dt}, that it pays to depositors. Interest rates in interbank market r_{Bt} generate revenues or costs depending on the net position of the bank. Rate on bonds are exogenous, and interest rates on bonds are assumed to follow a pure random walk process:

$$r^B_{t+1} = r^B_t + \varepsilon^B_{t+1} \tag{3}$$

$$\text{with} \qquad E[\varepsilon^B_t] = 0 \qquad E[\varepsilon^B_{t+i}\varepsilon^B_{t+j}] = \sigma^2_B \quad i = j, \quad E[\varepsilon^B_{t+i}\varepsilon^B_{t+j}] = 0 \quad i \neq j.$$

[8] A detailed study of the industrial costs of deposit is provided by Osborne (1982), and the results are consistent with this simplifying assumption.

[9] Wheelock and Wilson (2001), for example, find evidence of increasing returns to scale for small and medium size banks for the period following 1985, while the restriction of constant returns to scale could not be rejected for large banks. The finding of relevant return to scale is probably due to the progressive deregulation of the banking sector.

Reserves are assumed not to provide a return, so that reserve requirements are for the bank equivalent to a tax on deposits.

One of the most relevant functions of banks is to evaluate the risk of uncertain investment projects that other agents cannot price, because they lack the skills and the information necessary to attribute a probability distribution to the outcome of the investment. To undertake this activity banks must invest resources to obtain and process the relevant information, for example by monitoring their borrowers. The returns of the investment in information are thus decreasing, since the available stock of knowledge represents a binding constraint. This implies that banks cannot increase direct lending at will without reducing the efficiency of their monitoring and screening processes. Increasing direct lending indefinitely would mean that sooner or later they would finance investment projects of decreasing quality. As a consequence, default costs are assumed to be convex:[10]

$$D(L_t) = \frac{1}{2} v_t L_t^2, \qquad \text{where} \tag{4}$$

It is important to stress that in this formulation these are *expected* default costs and they are assumed to deterministic to keep the analysis simple. Although default costs are highly counter-cyclical and they are concentrated in periods of recessions rather than being spread over time, this assumption may not be over-restrictive. This for example is the case if the stochastic process of default costs displays a high degree of persistence: for standard values of discount rates, distant periods of time have a relatively small weight in the decision process, so that the cyclical behaviour of the economy may generate relevant changes in long-term expectations.

[10] Diamond (1991) has convincingly suggested that in an ideal world were banks could finance many different investment projects whose returns are i.i.d., they could eliminate delegation costs by means of diversification, by the law of large numbers. In the real world, where projects are not independent, some non-diversifiable risk remains. Furthermore, it may be difficult to achieve an extreme degree of diversification because of information costs; if part of the information costs are fixed, individual loans cannot be beneath a certain size to be profitable. Benefits from diversification are thus a concave function of the size of the portfolio. Agency costs among different layers of management, on the other hand, are likely to increase as the size of the bank grows since it becomes increasingly difficult to coordinate different agents. In this framework my assumption implies that agency costs grow non-linearly, as it is implicitly assumed by Berger and Udell (2002).

2.0.4 Deposit services

Households and firms make use of deposits not just as a financial asset for portfolio allocation, but mainly because banks provide them with transaction services. The provision of payment services, though, implies the establishment of mutual trust between bank and depositor, generating substantial search costs for depositors and the bank. Because of these costs, depositors do not easily switch from one bank to another when fees and interest rates are marginally changed. Flannery (1982) and Hess (1995) have provided substantial evidence of the empirical relevance of transaction costs (search costs in particular) in the market for deposits. Afterwards, deposits have been in fact increasingly described as quasi-fixed inputs.

In order to develop a simple as possible dynamic model, albeit somewhat unrealistically, I initially assume that interest rate on deposits are set competitively. This assumption will be relaxed in a final section. To highlight the properties of the model, I take interest rates on deposits as pure random walk processes, but the results can be extended to different stochastic processes:

$$r_{t+1}^D = r_t^D + \varepsilon_{t+1}^D \tag{5}$$

$$\text{with} \qquad E[\varepsilon_t^D] = 0 \qquad E[\varepsilon_{t+i}^D \varepsilon_{t+j}^D] = \sigma_D^2 \quad i = j, \quad E[\varepsilon_{t+i}^D \varepsilon_{t+j}^D] = 0 \quad i \neq j.$$

The demand for deposit services depends on nominal income, and the time series of income and prices have a strong persistence; I assume that real output and (albeit unrealistically) the price level are AR(1) processes, respectively:

$$Y_{t+1} = \gamma_Y Y_t + \varepsilon_{t+1}^Y = Y_t + g_Y Y_t + \varepsilon_{t+1}^Y, \tag{6}$$

$$P_{t+1} = \gamma_P P_t + \varepsilon_{t+1}^P = P_t + \Pi P_t + \varepsilon_{t+1}^P. \tag{7}$$

The income demand for nominal deposits is less than unitarily elastic:

$$ID_t^N = (Y_t P_t)^{1/\eta}, \qquad \eta > 1. \tag{8}$$

As a result of the behaviour over time of income and prices, in the next period, assuming that the correlation between the two error terms is zero:

$$E_t[ID_{t+1}^N] = E_t[(\gamma_Y Y_t + \varepsilon_{t+1}^Y)(\gamma_P P_t + \varepsilon_{t+1}^P)]^{1/\eta} = (\gamma_Y \gamma_P)^{1/\eta}[Y_t P_t]^{1/\eta} = (\gamma_Y \gamma_P)^{1/\eta} ID_t^N. \tag{9}$$

Expressed in real terms, the transaction demand for deposits is:

$$E_t[D_{t+1}] = \frac{ID_{t+1}^N}{P_t+1} = E_t\left[\frac{(\gamma_Y\gamma_P)^{1/\eta} ID_t^N}{P_{t+1}}\right] = \frac{(\gamma_Y\gamma_P)^{1/\eta} ID_t^N}{\gamma_P P_t} = \delta \frac{ID_t^N}{P_t} = \delta ID_t, \tag{10}$$

$$\text{with} \qquad \delta = \frac{(\gamma_Y\gamma_P)^{1/\eta}}{\gamma_P}. \tag{11}$$

The behavior over time of firm's demand for deposit services depends on aggregate expenditure. In addition, though, I assume that the amount deposits held by firms is influenced by the quantity of loans issued by the bank. In the case of small banks this is the case because banks compel firms to deposit a fraction of the loans they issue. In this way banks lock in borrowers, not allowing them to use competitors to manage their payments. Managing their payments, they keep an eye on the borrowers and earn the accompanying fees for transaction services. Sprenkle (1969, 1972) offered an explanation of this kind, while observing that it is impossible to justify the extent of the firms' deposits on the basis of inventory theoretic models. By means of these contractual agreements, bankers can monitor the liquidity of the borrowers in real time, obtaining the fundamental stream of information that allows them to evaluate and price the risk of the firms' investment projects. Compensating balances were the rule in the US during the 1960s and 1970s, while they are much less used today since banks rely to a large extent on commitment loans. Nevertheless they are still sizeable in the case of small banks.[11] In the case of large banks, the impact of loans issuance on deposits depends on the liquidity effect.[12] A bank operating in monopoly regime would not need to worry for the amount of deposits in its liabilities since all the sums lent would feed back as deposits. The bank would thus generate its liabilities by managing its assets, as it is the case for central banks. In an oligopolistic market, a large bank may rely on this feedback mechanism only partially, since the liquidity created is shared with the other banks of the system. Nevertheless, on average an amount of liquidity proportional to the bank's market share in the market for deposit will feedback from loans to deposits. This component of the demand for deposits depends on the amount of loans of the current period only:[13]

$$D_t^f = \kappa L_t. \tag{12}$$

The coefficient κ captures the effect of the feedback of loans on deposits. I make the further assumption that:

$$(1-q)\kappa > 1-\delta. \tag{13}$$

This assumption guarantees that issuing loans will raise deposits because of sufficient inertia in the exogenous component of the demand for deposits. It is not very restrictive since $1-\delta$ is always very small, as can be easily verified, and the value of q is also small (0.1 on average in the US).

Summing deposits of firms and households the expected level of deposits of the bank becomes:

$$E[D_{t+1}] = \delta D_t + \kappa E[L_{t+1}] \tag{14}$$

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[11] Berger and Udell (1995) provided evidence that 7 per cent of small commercial borrowers faced compensating balance requirements.

[12] See Diamond and Rajan (2001a,b, 2002, 2005).

[13] This assumption is necessary in order to make the model tractable. But it can be justified considering that the lag in the operation of the feedback should not be too long: in general most of the portfolio of retail banks is made up of short-term loans. On the other hand, if current deposits are assumed to depend on loans of the previous period the results do not change in a relevant way.

In short, the behavior over time of the demand for deposit services follows the transaction demand form money and displays the same inertia, but banks influence this dynamic issuing loans.

2.0.5 The demand for loans

The cost of information generates monopoly power in the market for loans.[14] Relationship lending in fact allows the bank to price monopolistically, and the higher return due to the market power makes the higher risks of the project worthwhile.[15] Consequently I introduce in the problem of the bank a stochastic demand relationship for loans:

$$L_t = a - br_t^L + dr_t^B + \eta_t, \tag{15}$$

where η_{t+1} is a white noise error term.[16]

3 Solution

3.1 Intertemporal maximization

The banking firm maximizes its expected profits over an infinite horizon period. The problem, for every pair of positive real numbers (v, u), can be expressed as:

$$Max \quad V = \sum_{t=0}^{\infty} \beta^t E\left[r_t^L L_t + r_t^B F_t - r_t^D D_t - \frac{1}{2} v_t L_t^2 - u_t D_t + f D_t - z L_t\right],$$

$$w.r.t. \qquad \{F_t, D_t\}_0^{\infty}, \qquad\qquad s.t.$$

[14] The existence of intra-industry monopoly power in the banking industry of the US has been empirically confirmed by Cosimano and McDonald (1998).

[15] In the model by Sharpe (1990), by establishing long-term relationships with its customers, a bank learns more than others about the business and the capability of the borrower. This information asymmetry generates a rent that allows banks to finance risky projects whose information is very opaque, which cannot be financed in the market. Establishing the relationship and developing their knowledge, banks provide a valuable service, they create the knowledge necessary to price the risk. The price that firms pay for this service is the monopolistic rent that they pay on loans.

[16] For an analysis of the factors that affect the intercept term of a linear demand curve for loans, see Bertoni et al. (1975).

$$L_t + F_t + R_t = D_t + NW, \qquad R_t = qD_t, \qquad L_t = a - br_t^L + dr_t^B + \eta_t,$$

$$E[D_{t+1}] = \delta D_t + \kappa E[L_{t+1}]. \tag{16}$$

The logical structure of the profit function is very simple: revenues come from the interest rate spreads, the costs that must be deducted are the cost functions, as previously defined. The discount factor is $\beta^t = \frac{1}{(1+r)^t}$, where r, is the banker's discount rate. Deposits are the state variable of the problem, while F_t, the net position in the interbank market, is the control variable of the bank.

3.1.1 The dynamic constraint

Some features of the model are standard: the demand function for loans, in particular, solves for the interest rate on loans, and its value is substituted in the profit function. The main peculiarity of the model lies in the deposit demand schedule, because its presence makes stocks relevant, and the model becomes dynamic. To understand why, the equation can be solved for the quantity of loans. From:

$$E[D_{t+1}] = \delta D_t + \kappa E[L_{t+1}]. \tag{17}$$

the following obtains:

$$L_t = \frac{1}{\kappa}\Big\{E[D_{t+1}] - \delta D_t\Big\}. \tag{18}$$

Substituting this function for L_t in the profit function, I can observe that the quadratic cost on loans works as a quadratic adjustment cost on deposits.

In its dynamic properties, the model is very close to an investment model. Deposits are the state variable, and play the role of capital. The bank can increase the stock of deposits issuing loans, so that the quantity of loans is akin to the level of investment. It is important, however, to view the net position in the interbank market, F_t, as the control variable, so that loans are obtained residually. This brings out the simultaneous nature of the solution for the optimal size and the optimal composition of the portfolio.

3.1.2 Euler equation

After some manipulations, the following difference equation can be obtained from the Euler equations solving the problem:

$$\begin{aligned} E\left[F_{t+1}\right] = & \frac{1-(1-q)\kappa}{\delta\beta}F_t + \frac{(1-q)\left\{\delta^2\beta - [1-(1-q)\kappa]\right\}}{\delta\beta}D_t + \\ & + \frac{\beta\delta + (1-q)\kappa - 1}{\delta\beta}NW + \frac{1-(1-q)\kappa - \delta H}{\delta\beta}E\left[\frac{1}{\alpha}Z_{t+1}\right], \end{aligned} \tag{19}$$

where:

$$(bv+2)/b = \alpha \qquad \varepsilon_t^L = \frac{1}{b}\eta_t, \tag{20}$$

and

$$\begin{aligned} Z_{t+1} = & \left[\left(1-\frac{d}{b}\right)(\delta\beta - L) + (1-q)\kappa L\right] r_{t+1}^B - (\delta\beta - L)\varepsilon_{t+1}^L \\ & + \kappa\left[r_t^R q - r_t^D - u + f\right] + (\beta\delta - 1)\left(z - \frac{a}{b}\right), \end{aligned} \tag{21}$$

Equation (19) together with the original dynamic constraint, given by the demand condition, (which I rewrite after substituting the budget constraint) form a system of difference equations:

$$\begin{aligned} E\left[D_{t+1}\right] = & \frac{\delta}{1-\kappa(1-q)}D_t - \frac{\kappa}{1-\kappa(1-q)}E\left[F_{t+1}\right] + \\ & + \frac{\kappa}{1-\kappa(1-q)}NW. \end{aligned} \tag{22}$$

3.2 Stability conditions

The solution of this class of dynamic models is normally obtained as a function of the roots of the system, which are in general quite complicated. But because of the simplicity of the structure of the model, there is a closed form solution for the eigenvalues of the system that allows studying its stability. The eigenvalues are:

$$\frac{1}{\delta} \qquad \text{and} \qquad \beta\delta. \tag{23}$$

If one of the two roots is larger, while one is smaller than one, it is possible to solve the model partially forward and partially backward, and obtain a saddle-path equilibrium. This implies the existence of a unique convergent trajectory, on which the rational expectations equilibrium lies. Necessary and sufficient condition are:

$$\begin{cases} \delta > 1 & \text{and} \qquad \beta\delta > 1 \\ \delta < 1 & \text{and} \qquad \beta\delta < 1 \end{cases}$$

In order to understand these conditions I must recall the expression for δ, from Equation (11):

$$\delta = \frac{(\gamma_Y\gamma_P)^{1/\eta}}{\gamma_P} = \gamma_Y^{\frac{1}{\eta}}\gamma_P^{\frac{1-\eta}{\eta}}. \tag{24}$$

Thus $\delta < 1$ implies $\gamma_Y < \gamma_P^{\eta-1}$. The income demand for deposits can grow at a faster rate than prices, but it must not be of exponential order higher than $\eta - 1$. This insures that the demand converges to a finite value as the time horizon tends to infinity. In the remainder of the work I will assume that the condition $\delta < 1$ holds.

When this condition is satisfied, the condition regarding the other eigenvalue is satisfied *a fortiori*, since β is a discount factor. Interestingly, it then follows that the dynamic of system is not influenced either by the cost coefficients, or by the feedback process (neither would it be by reserve requirements). This is due to the particularly simple structure of the model, wherein costs on deposits are linear. But even in more complex models, with more non-linear aspects, the stability of the system would depend fundamentally on the same two variables as here: the discount factor and the coefficient of the lagged term in the deposit demand condition (which, in turn, depends on the income demand for deposits).

In order to guarantee the stability of the system, a typical transversality condition must also be satisfied.

3.3 Rational Expectations Equilibrium

The Rational Expectations Equilibrium of the system follows through substitution. Substituting Equation (19) in Equation (22), I obtain a second order difference equation relating to the stock of bonds:

$$E[F_{t+1}] - \left[\frac{1}{\delta\beta} + \delta\right] F_t + \frac{1}{\beta} F_{t-1} = \frac{[1-\delta]\,[\beta\delta - 1]}{\delta\beta} NW + \\ -E\left[\frac{1}{\beta\alpha}\right] Z_t + E\left[\frac{1-(1-q)\kappa}{\delta\beta\alpha} Z_{t+1}\right]. \tag{25}$$

Following the same procedure, I can write the value of D_t as:

$$E[D_{t+1}] - \left[\frac{1}{\delta\beta} + \delta\right] D_t + \frac{1}{\beta} D_{t-1} = -\frac{\kappa}{\delta\beta} E\left[\frac{1}{\alpha} Z_{t+1}\right]. \tag{26}$$

Using the expectation lag operator H, such that $H^{-j}E_{s-1}x_s = E_{s-1}x_{s+j}$, the left hand side of the equation can be expressed as:

$$E[F_{t+1}] - [\frac{1}{\delta\beta} + \delta] F_t + \frac{1}{\beta} F_{t-1} = (1-\lambda_1 H)(1-\lambda_2 H) E[F_{t+1}]. \tag{27}$$

Where λ_1 and λ_2 are the reciprocal of the roots of the system. The right hand side can be rewritten as:

$$1-(\lambda_1+\lambda_2)H+\lambda_1\lambda_2 H^2, \qquad \text{so that:}$$
$$-(\lambda_1+\lambda_2)=\frac{1}{\delta\beta}+\delta \qquad \text{and} \qquad \lambda_1\lambda_2=\frac{1}{\beta}. \tag{28}$$

Thus:

$$\lambda_1=\delta \qquad \lambda_2=\frac{1}{\beta\delta}. \tag{29}$$

Next, Equation (25) can be rewritten as:

$$(1-\lambda_1 H)F_{t+1}=\frac{1}{(1-\lambda_2 H)}E_t\left\{\frac{[1-\delta]\,[\beta\delta-1]}{\delta\beta}NW+\frac{1-(1-q)\kappa-\delta H}{\delta\beta\alpha}Z_{t+1}\right\}. \tag{30}$$

From Equation (21) the constant terms can be isolated, obtaining:

$$Z'_{t+1}=\left[\left(1-\frac{d}{b}\right)(\delta\beta-H)+(1-q)\kappa H\right]r^B_{t+1}-(\delta\beta-L)\varepsilon^L_{t+1}-\kappa r^D_t, \tag{31}$$

and

$$C=(1-\beta\delta)\left[\frac{a}{b}-z\right]-\kappa(u-f). \tag{32}$$

Assuming that $\lambda_1<1$ and $\lambda_2>1$, the right-hand side can be solved forward, applying the algorithm developed by Sargent (1979). The transversality condition is satisfied whenever interest rates are bounded processes, and Equation (25) can be solved as:

$$F_{t+j+1}=\delta F_{t+j}+(1-\delta)NW-\frac{1-(1-q)\kappa-\delta H}{\beta\delta}\sum_{i=1}^{\infty}\left(\beta\delta\right)^i E_{t+i}\left[\frac{Z'_{t+j+i+2}}{\alpha}\right]+$$
$$-\frac{1-(1-q)\kappa-\delta}{(1-\beta\delta)\alpha}C. \tag{33}$$

3.4 Composition of the portfolio

Loans

The rational expectation equilibrium quantity of loans can be easily obtained from the budget constraint $L = (1-q)D - F + NW$, after obtaining the equilibrium values of deposits and bonds. The value is:

$$E[L_{t+j+1}] = \delta L_{t+j} + \delta NW + \frac{1-\delta H}{\delta\beta}\sum_{i=0}^{i+1}\left(\delta\beta\right)^{i} E_{t+i}\left[\frac{Z'_{t+j+i+1}}{\alpha}\right] + \frac{1-\delta}{(1-\delta\beta)\alpha}C, \qquad (34)$$

Under the assumption that interest rates follow a random walk process, this expression becomes:

$$L_{t+j+1} = \delta L_{t+j} + \delta NW + A_2 r^B_{t+j+1} - \frac{(1-\delta)\kappa}{(1-\delta\beta)\alpha} r^D_{t+j+1} - \frac{z}{\alpha} - \frac{\kappa(u-f)}{(1-\delta\beta)\alpha} + \frac{a}{b\alpha}. \qquad (35)$$

where

$$A_2 = \frac{(1-\delta)\left[\left(1-\frac{d}{b}\right)(\delta\beta-1) + (1-q)\kappa\right]}{(1-\beta\delta)\alpha}. \qquad (36)$$

Assumption (13) guarantees that A_2 is positive.

The equilibrium quantity of loans is a function of its lagged value and of the expected future values of a set of variables. These variables are: the quantity of net worth; the current and lagged values of interest rate on bonds, and interest rate on deposits; the coefficients of the industrial costs; the revenues from fees; the intercept of the demand for loans. All of these factors have the expected sign, and their interpretation is the same in most regards as it would be in a static monopolistic model. So the equilibrium quantity is an increasing function of aggregate demand, as indicated by the intercept of the demand curve, and a decreasing function of all cost terms, while it grows with the revenues from fees. I will say something about the omitted influence of net worth in a separate section.

Loans are a decreasing and convex function of v, as long as the intermediation margin is positive. The equilibrium quantity of loans is thus larger the lower the value of expected default costs. A fundamental aspect of this model is that expected default costs, by influencing the equilibrium issuance of of loans, constrains the size of the portfolio of the bank, since by issuing loans the bank affects the optimal quantity of deposits too.

In line with the intuition, the main factor on which the equilibrium quantity depends are the coefficients of the demand curve. Consider first the importance of market power in my dynamic framework. As noted in Equation (20) before:

$$\alpha = \frac{bv+2}{b}, \tag{37}$$

It is then easy to verify that the solution depends in a fundamental way on the coefficient b, which measures the interest rate sensitivity of the demand for loans and gives the slope of the demand curve. Examination of Equation (35) shows that most terms of the solution, interest rates and costs in particular, are multiplied by $1/\alpha$, while the intercept of the demand curve is multiplied by $1/b\alpha$. This makes a big difference since $1/\alpha$ is an increasing function of b, while $1/b\alpha$ is a decreasing function of b. In fact, when the value of b is not large, the main positive influence on the equilibrium is the intercept of the demand curve. Otherwise (if the elasticity of the demand for loans is large), the main positive influence is the interest rate on bonds. This is easily understood since:

$$\frac{1}{\alpha} = \frac{b}{bv+2} \begin{cases} \lim_{b\to 0} = 0 \\ \lim_{b\to\infty} = \frac{1}{v} \end{cases} \qquad \frac{1}{b\alpha} = \frac{1}{bv+2} \begin{cases} \lim_{b\to 0} = \frac{1}{2} \\ \lim_{b\to\infty} = 0 \end{cases}$$

It follows that as competitive pressures increase, the relevance of the intercept of the demand curve proportionally declines, and the issue of loans increasingly depend on the margin between the interest rate spread and industrial costs. On the contrary, whenever the market power is relevant because the demand is inelastic, demand side factors are decisive. This is likely to be the case for small firms lending.

The equilibrium quantity of loans is an increasing function of the interest rate on bonds. The effect of interest rates on bonds on the issue of loans depends on two different factors: the interest rate sensitivity of the demand for loans, which is positive,[17] and the direct effect on banks, the supply side of the market. The supply of loans is affected by the rate on bonds in two different, contrasting, ways. The first is a standard negative portfolio composition effect (analogous to that in static analysis): the rate on bonds is opportunity cost on loans. The second is a positive effect, which is due to the feedback process linking the size of the portfolio to the issue of loans. Higher rates in fact increase the return of both components of the assets portfolio, loans and bonds. This represents a positive incentive for the issue of loans, which always dominates the negative opportunity-cost direct effect, based on Equation (13).[18] Thus, the equilibrium quantity in the market for loans is an increasing function of the interest rates on bonds, independently of the expected level of the default cost, and independently of the impact of the interest rate on bonds on the demand for loans.

The level of the interest rate on deposits affects the issue of loans negatively. The rate on deposits is in fact a cost that the bank has to face in order to issue loans and it reduces proportionally the profitability of loans. For the same reason industrial costs have a negative impact too, while, on the contrary, marginal revenues from transaction services have a positive influence, because the bank makes economies of scope between the two services.

3.4.1 Bonds

The amount of resources invested in bonds or lent in the interbank market represents a buffer that the bank manages in order to optimize the issuance of more profitable loans. The size of this buffer depends on the level of interest rate spreads and the marginal costs of loans.

The general solution for the equilibrium quantity of bonds was shown in Equation

[17] This effect is amplified by the market power of the bank.

[18] It can in fact be seen that the first of the three terms in the curled bracket that multiplies r^B_{t+j} is always larger than one (which is the value of the second), thanks to my basic assumption of Equation (13).

(33). Following the assumption that interest rates obey a random walk process, the solution can be expressed as:

$$F_{t+j+1} = \delta F_{t+j} + (1-\delta)NW - \frac{1-(1-q)\kappa-\delta}{(1-\beta\delta)\alpha}\left\{(1-\beta\delta)\left[\frac{a}{b}-z\right]-\kappa(u-f)\right\}+ \\ +A_1\left(r^B_{t+j+1}-\kappa r^D_{t+j}\right), \quad (38)$$

where

$$A_1 = \frac{[1-(1-q)\kappa-\delta]\left[\left(1-\frac{d}{b}\right)(1-\delta\beta)-(1-q)\kappa\right]}{(1-\beta\delta)\alpha}. \quad (39)$$

Assumption (13) guarantees that the sign of A_1 is positive. As a consequence, the bank keeps a buffer that is larger the bigger the spread between the market interest rate and the rate on deposits. On the contrary, the higher the marginal costs on both loans and deposits, the lower the size of the buffer, while it grows with the intercept of the demand for loans (depending largely on aggregate demand).

When interest rate spreads are healthy, bonds and loans are complement in the portfolio of the bank. A situation where the bank has a negative buffer, implying that the bank borrows from the interbank market or from financial markets, in order to have more loans than deposits, arises only when interest rates are very low with respect to marginal costs. Thus a protracted expansionary monetary policy pushes the bank to rely on market finance to increase direct lending. Relaxing the assumption of perfect competition in the market for deposits would strengthen the result, since the interest rate on deposits would be set as a mark-down on the rate on bonds. The rationale behind this behavior is that low interest rates make the buffer very costly. When the spread falls below a certain threshold, not only it is too expensive to hold a buffer, but leverage must rise to keep profitability, since industrial marginal costs are constant. For a given level of expected default costs, profit margins depend on the difference between interest rate spreads and marginal industrial costs. By borrowing in financial markets, to the extent that interest costs do not rise substantially, as it is the case for large banks that implicit benefit from state guarantees, the bank can increase returns because market debt does not incur in industrial costs. When interest rates are particularly low, market finance becomes cheaper because deposits face both industrial costs and increasing deposits rates when they are priced monopolistically (as it is normally the case). The higher returns from market finance, though, should be traded off with the increased liquidity risk.

Default costs shrink the size of the forward-looking part of the equation, reducing the size of the portfolio even in the case of bonds. Default costs in fact reduce the optimal quantity of loans and, as a consequence, the size of the whole portfolio.

3.4.2 Deposits

The general solution for deposits is the following:

$$E\left[D_{t+j+1}\right]=\delta D_{t+j}+\frac{\kappa}{\beta\delta}\sum_{i=1}^{\infty}\left(\beta\delta\right)^{i}E_{t+i}\left[\frac{Z'_{t+j+i+2}}{\alpha}\right]+\frac{\kappa}{\alpha(1-\beta\delta)}C. \tag{40}$$

When interest rates obey a random walk process, it follows that:

$$D_{t+j+1}=\delta D_{t+j}+A_3 r^B_{t+j+1}-\frac{\kappa^2}{(1-\beta\delta)\alpha}r^D_{t+j+1}-\frac{\kappa\left\{(1-\beta\delta)\left[z-\frac{a}{b}\right]+\kappa(u-f)\right\}}{(1-\beta\delta)\alpha}. \tag{41}$$

where

$$A_3=\frac{\kappa\left[\left(1-\frac{d}{b}\right)(\delta\beta-1)+(1-q)\kappa\right]}{(1-\beta\delta)\alpha}. \tag{42}$$

Assumption (13) guarantees that the sign of A_3 is positive.

The intertemporal equilibrium level of deposits depends negatively on the costs of both deposits and loans. The level of deposits grows with the own interest rate, as a function of the sensitivity of the demand for deposits. The lagged value of the own rate has a negative impact.

The sign of the equilibrium quantity of deposits as a function of contemporaneous interest rate on bonds is positive, since the size of the portfolio depends on the issue on loans; the profits from lending, in fact, grow with the interest rate on bonds, and thus the interest rate has a positive influence on the equilibrium level of deposits. The standard substitution effect due to the decreased demand for money when interest rate increase here is captured by the induced variation of the rate on deposits.

These results provide a theoretical rationale for the empirical evidence provided by Chari et al. (1995) that M1 has a positive correlation with future values of the interest rate, while the correlation with contemporaneous and past values is negative. Because of the assumption that future interest rates follow a random walk (and accordingly the deterministic component is expected to remain constant), in my formulation, the contemporaneous interest rate synthesises the effect of both future and contemporaneous rates. But it can easily be seen from the general solution [19] that the negative demand side effect exclusively affects the contemporaneous value. On the contrary, the equilibrium level of deposits is an increasing function of all future expected values of the interest rate on bonds. This supply side effect is normally neglected in monetary theory, but can explain the limited interest rate elasticity of the demand for money that is found in empirical studies. Usual estimates of the demand for money implicitly assume an infinite elasticity of the supply of deposit services. But this apparently reasonable assumption is only innocuous in the short run. In the long run, the supply of deposit services, together with the size of banking intermediation, necessarily depends on the profits of the industry. Thus, the negative interest rate elasticity of the demand for money in the long run will be limited by positive responses of the yields on deposits.

Default costs shrink the size of the forward-looking part of the equation, reducing the size of the portfolio. These costs are the true constraint on the size of the portfolio, thus putting a limit to the liquidity creation. The positive correlation between shocks in the demand for loans and default costs reduces the equilibrium level of deposits still further.

[19] Equation (41).

3.4.3 The interest rate on loans

The interest rate on loans can be obtained by substituting for the quantity of loans, both current and lagged, in Equation (35), the value obtained from the demand for loans, Equation (15). The solution can be expressed as:

$$r^L_{t+j+1} = \sigma_0 + \sigma_1 r^L_{t+j} + \sigma_3 r^D_{t+j+1} + \sigma_5 r^B_{t+j+1} + \sigma_6 r^B_{t+j}, \tag{43}$$

where

$$\sigma_1 = \delta, \qquad \text{and} \qquad \sigma_5 = \frac{d}{b} - A_2.$$

The interest rate on loans thus displays the same degree of persistence as the demand for money. Since δ is normally quite large, changes in the interest rate on loans set by this bank are very sluggish.

The impact of market interest rate shocks is ambiguous. The standard demand side effect is positive if bonds and loans are substitutes for borrowers, but it is at least partially offset by a supply side effect produced by the forward looking impact of market interest rates on loans. Since higher market rates push the bank to increase the issuance of loans, this effect pushes down the equilibrium interest rate on loans. There is a strong presumption that the supply side effect is dominated by the demand side, so that higher market rates drive interest rates on loans higher. Nevertheless, this supply side effect has the important implication that the bank of the model always smoothes interest rate shocks. This result is in line the empirical available evidence and with the result of Fried and Howitt (1980). In this case, however interest rate smoothing is not the result of implicit insurance contracts, but it simply derives from the forward looking behavior of the bank. And, contrary to Fried and Howitt (1980), this result does not depend on market power: as the interest rate semi-elasticity of the demand for loans grows, the impact of the supply side effect becomes larger.

If the intercept of the demand for loans is assumed to depend on aggregate demand, an assuming that aggregate demand follows an AR(1) process, the solution becomes:

$$r^L_{t+j+1} = \sigma_0 + \sigma_1 r^L_{t+j} + \sigma_3 r^D_{t+j+1} + \sigma_5 r^B_{t+j} + \sigma_6 r^B_{t+j+1} + \sigma_7 Y_{t+j+1} + \sigma_8 Y_{t+j+1}, \qquad (44)$$

where $\sigma_7 = \frac{1}{b}\left[g - \frac{1}{b\alpha}\right]$. An increase in aggregate demand produces a demand side effect g that drives the rate on loans up and an offsetting effect from the supply side, since the equilibrium quantity of loans becomes larger.

3.5 Extensions and limitations

3.5.1 Monopolistic pricing of deposits

Salop (1976); Salop and Stiglitz (1977, 1982) suggest that search costs allow firms to charge non-competitive prices. Given the relevance of search costs in the market for deposit services, each bank may be assumed to have some monopoly power in this market (monopsonistic power in the market for deposits), as confirmed by the empirical analysis by Hannan and Berger (1991) and Neumark and Sharpe (1992). Assuming a standard demand curve for deposit services (supply of deposits from the public) as:

$$D_t = c + e r^D_t - h r^B_t,$$

the intercept term of Equation (40) and (40) remains identical, the only difference being that instead of r^D_t the new solution has the value of $\theta_t = -\frac{c}{e} + \frac{h}{e} r^B_t$. The positive impact of the market interest rate on the equilibrium value of deposits is now smaller than in the competitive case, as intuition would suggest, since the bank must now bear an interest cost to increase deposits. However it is easy to realize that the impact of market rates on the equilibrium quantity of deposits is still positive.

By substituting from Equation (45) into Equation (40) for both periods t and $t+1$, the interest rate on deposit can be obtained, and it has the same kind of formal structure as that on loans. The result simply implies that the bank sets deposits interest rates as mark down on the interest rate on bonds. The only complication here, with respect to a static model, is that now higher market interest rates imply a higher equilibrium quantity of deposits, so that the direct positive impact of an increase of market interest rates on the rate of deposits due to standard substitution effect, is now amplified in proportion to the value of A_3. In this dynamic framework deposits are necessary to lend, and it is implicitly costly to adjust the stock, consequently the bank is willing to benefit of a lower mark-down on the rate it pays depositors today, in order to increase future profits.

The solution of this problem, however, is slightly more complicated than in the competitive case because, while before the roots of the solution for deposits and bonds were the same, this is not the case here. The left hand side of difference equation for deposits now becomes:

$$E[D_{t+1}] - \left[\frac{1}{\delta\beta} + \delta + \frac{\gamma\kappa(1-q)}{1-[\kappa-(1-q)]\delta\beta}\right] D_t + \frac{1}{\beta} D_{t-1},$$

where $\gamma = \frac{2}{e}$ captures the impact of the semi-elasticity of the demand for deposit services. It can be shown that the roots of equation are one smaller and one bigger than one, so that the solution is saddle-path stable.[20] The two roots, λ_1 and λ_2 are different from those of the solution for F_t that were the same as those for D_t in the competitive case, namely $\frac{1}{\delta}$ and $\beta\delta$. In particular, the backward looking roots for deposits is now bigger, implying that deposits have a higher degree of persistence and are less forward looking, since changing the stock of deposits in this case implies an interest cost that is absent in the competitive case.

3.5.2 Oligopolistic competition

The model could easily (at lest in abstract) be modified to a Cournot model, without altering the main results. The problem of every individual bank in this case includes the

[20]See Sargent (1979) p.197-198.

market share as an unknown of the problem, and it takes into account the result of the same optimisation problem performed by the others banks. There would now be n firms facing the respective n maximization problems, that include the problems of the competitors in the price setting equation. And each individual firm's problem now includes as an unknown the value of the market shares $\psi = \frac{L}{L_j}$ and $\chi = \frac{D}{D_j}$. The n equations would provide the optimal supply functions. The condition of aggregation of the loan and deposits supply schedules provides the two extra equations that allow closing the system:

$$L = \sum_{j=1}^{n} L_j, \quad D = \sum_{j=1}^{n} D_j, \tag{45}$$

where n is the number of firms. The main implication of this more complicated model is that in each bank problem the expectations of the competitors play a relevant role. A bank's deposits now grow not only with the bank's own loans but even as other banks increase lending. Thus a conservatively managed bank might end up reducing new lending even if it does not expect higher default costs, when it expects that other banks' larger expected default costs would reduce the overall amount of liquidity in the system.

3.5.3 Stochastic default costs

The default cost function is assumed to be deterministic to keep the model tractable. It is possible to relax this assumption, by considering the coefficient of the default cost function to be time-varying and stochastic, and the equilibrium solutions do not change in a substantial way. In this case all the equilibrium solutions are reduced by the covariance between interest rates and default costs. However in this case the certainty equivalence property does not hold, because the choices of the bank influence the probability distribution for the optimal decision of the bank. It is possible to overcome this problem by studying it in the context of an optimal experimentation problem, as suggested by Cosimano (2003). However this method requires numerical techniques to get the solutions and such an analysis, although more general and rigorous, would need detailed data on parameter values that are not easily obtainable.

3.5.4 Capital ratios

The main limitations of the model is the lack of a proper analysis of the capital structure of the bank, net worth was in fact assumed to be constant. This limitation can only partially be addressed in this framework, since the analysis treats loans as a portfolio, whose risk depends on expected default costs that are not modelled. The model can only highlight the mechanics implicit in a legal requirement of a minimum ratio between capital and loans, *when the legal requirement binds*.[21] If the net worth of the bank has to cover at least a fixed proportion of the loans issued, the budget constraint becomes the following:

$$(1-\theta)L_t = (1-q)D_t - F_t. \tag{46}$$

It can be easily seen that in this case the results of the model would change in a simple way. The term in net worth obviously disappears, and in the final result both intercept terms are multiplied for $1-\theta$. The effect of this legal requirement is to reduce the forward looking part of the solution. As a consequence, in this case the size of the portfolio is reduced in proportion to the legal requirement coefficient, while the composition of the assets portfolio is not affected. Clearly the amount of deposits is reduced as well, because deposits positively depend on the main spread Z_t. The impact of the imposition of capitalisation coefficients is similar to the effect of reserve coefficients on deposits, but it is much stronger. This result explains why in order to control the growth of monetary aggregates in periods of high inflation, the introduction of constraints on the issue of loans is effective, while the increase of reserve coefficient may not.

This treatment of capital requirements, though, is not satisfactory, because the stock market remains outside the analysis. I am, in fact, explicitly analysing conditions under which the Modigliani-Miller theorem does *not* hold. The explicit introduction of the stock market would be quite complex, and it is far beyond the scope of this work.

[21] A general treatment of the problem can be found in Kopecky and Hoose (2006).

4 Implications

4.1 Interest rate shocks

The bank described in this model smoothes the impact of interest shocks to its borrowers. The interest rate on loans set by the bank, in fact, changes less than proportionally as market interest rates vary. In this model, a permanent interest rate shock induces the bank to increase the size of the portfolio by issuing more loans. Higher interest rates on bonds, in fact, generate a larger demand for loans because loans and bonds are substitutes for borrowers, and they raise the supply of loans because loans become more profitable for the bank. The impact on the quantities is thus always positive. Moreover, the larger issuance of loans produces a higher supply of deposits services, so the issue is partially self-financed. When rates on deposits are set monopolistically, the higher level of deposits requires higher rates on deposits, and this limits the benefits of such a policy. The higher rate on bonds, in addition, reduces the demand for deposit services, further increasing the rate on deposits, although less than proportionally, since the bank sets the rates as a mark down on the market rate. If the cost of deposits becomes too high the bank may choose to finance lending by reducing the amount of bonds held as a buffer. If so, the bank finances the increased issue of loans by switching away from bonds. However, bonds and loans are generally complementary assets in the portfolio and the banks increases the buffer as loan issuance rises. Furthermore, because of convex default costs and a downward sloping demand curve, the bank faces decreasing returns to scale on loans, while returns to scale on bonds are constant. This too contributes to the buffer role of bonds, which rise and fall largely as a function of the dominant bank concerns with the issue of loans. The equilibrium quantity of bonds therefore is more volatile than the quantity of loans.

Lower interest rates following an expansionary monetary policy push the bank to reduce the buffer represented by the stock of bonds purchases or interbank lending, since the return on these assets becomes lower. If the low interest rates drive the interest spread to a low enough level, then the bank has a strong incentive to finance the issuance of loans by borrowing in the interbank market or in financial markets. This is particularly the case for large banks that, explicitly or implicitly, benefit from state guarantees, since marginal interest costs on market borrowing are constant, while those on deposits are not, whenever deposits are priced monopolistically. Moreover, the provision of deposit services, although generating fees from payment services, implies relevant industrial costs. As a consequence, when interest rate spreads are compressed, market finance becomes much cheaper, since market liquidity risk is normally not priced in financial markets.

4.2 Real shocks

Unexpected sharp movements of relative prices generate shocks that affect borrowers, producing variations of current and expected default costs. Any shock which permanently raises default costs shrinks the size of the whole portfolio, since assets and liabilities decline proportionally. Consequently the model predicts that banks do not smooth permanent real shocks by creating liquidity. On the contrary, the bank of the model reacts to the higher default costs with a credit crunch, and by raising the interest rate charged on loans, amplifying the impact of the shock.

The behavior of the bank is not much different when the banker expects the real shock to be transitory, because the equilibrium quantity of loans is a convex function of default costs. Consequently, the equilibrium quantity is an increasing function of the variance of the shocks. The bank benefits from the higher variance because it can price monopolistically and the mark up is non-linear. If a negative real shock affects borrowers but is reversed in a second period, bank profits soar. The increase of the interest rate on loans charged when default costs grow is in fact larger than the decrease following the reversal of the shock. Consequently, this bank does not smooth any temporary shock providing insurance.

These results require some qualifications. In general, when heavy shocks hit the economy, it may become very difficult for the less informed lenders in the market to properly price certain risks, and the bond market may react abruptly to the shocks. In some cases the market may altogether dry out for some borrowers, because of the emergence of a lemon problem. But if banks are better informed, they may reduce their lending much less than the bond market. In this case banks may provide insurance even against permanent real shocks. Besides, since some borrowers are pushed to rely exclusively on banks, the demand for loans may surge as bonds become unavailable to finance certain risky projects. A strong enough increase of the demand could, in principle, push the bank to lend more, since the demand would in this case grow more than proportionally as default costs rise.

Large injections of capital, although providing an important buffer, do not increase significantly the incentive to lend of the bank of this model. In order for the optimal portfolio to increase, for a given level of demand and marginal industrial costs, either interest rate margins must grow, or expected default cost must decline. But to compensate for a twofold increase of default costs, interest margins must double, implying either much lower rates on deposits, or much higher rates on loans. To the extent that the banking system remains stable, any increase in risk aversion is matched by large inflows of deposits that reduce interest rates. This mechanism thus generates, at least partially, the incentives to lend. But if the stability of the banking system is challenged, so that the increased demand for monetary assets excludes deposits, than the credit crunch worsens generating a generalized run on the system. Moreover, in situation of stress, banks with a better quality portfolio gain market share, and the flight to quality can easily jeopardize the chances of survival of the weaker ones. If the system is characterized by regional banks, the weaker ones are those operating in the regions that are most affected by the negative shock. This mechanism thus amplifies the asymmetric impact of real shocks among different regions. Relevant economies of scale, in this framework, stem from diversification, as in Diamond (1984). If large banks can achieve a higher degree of diversification than smaller ones, then their expected average default costs are smaller. In this case their optimal portfolio is proportionally larger, and diversification implies more risk-taking. A straightforward implication is thus that large interregional banks may very important to smooth the impact of negative real shocks, when relative price variations have a diverging impact in different regions.

4.3 Investment banks and the credit crunch

The model highlights that the size of the optimal portfolio depends on the level of expected default costs. If for some reason the bank may presume its loan portfolio not to be subject to convex default costs, it will be tempted to expand the portfolio indefinitely.

Although this model describes a typical commercial bank, it can be used to understand the incentives that investment banks in the US where facing in recent years. Firms in the US allocate nowadays a substantial amount of their monetary assets to Money Market Mutual Funds that often provide a higher return than bank deposits. These higher returns have been obtained in recent years by purchasing risky commercial paper issued by corporations and banks, and asset backed commercial paper, rather than risk free assets. Most MMMF are managed by banks, and investment banks in particular. By managing MMMFs, investment banks can monetize assets such as mortgages or even corporate loans. Investment banks, in fact, can originate loans, for example financing leveraged buy-outs, or mortgages by means of specialized subsidiaries, or can purchase these risks in loan sale markets. By securitizing loans and mortgages, investment banks are assumed to distribute the risks to the rest of the market. But if a MMMF managed by an investment bank purchases assets securitized by the same bank, the process described is a process of liquidity creation in principle analogous to that of commercial banks. The only difference is that investment banks need the intermediation of financial markets. This carries a danger, since when heavy shocks hit the economy financial markets are subject to radical uncertainty and they may collapse because of the emergence of a ‘lemon’ problem. However, this process of liquidity creation is in principle not different from that of commercial banks, so that even in this case an increase in leverage generates larger and larger amounts of deposits. Moreover, since MMMFs where assumed to hold only risk free assets, they have remained almost free from regulatory constraints, such as reserve requirements.

By providing a large collateral in the securitized assets, investment banks have generated a large amount of securities that were treated as almost risk free loans. Given that the liquidity generation process took place by means of these assets, they where behaving like the representative bank of the model under the assumption of an extremely small expected default coefficient. The optimal portfolio was consequently extremely large. The collapse in the value of collateral, due to the decline in real estate prices, has generated a proportional increase in expected long-term default costs. Such a a heavy shock has generated a persistent increase in these costs that implies a much smaller portfolio.

5 Conclusion

When negative shocks of real nature affect borrowers, the bank described in this model does not provide insurance; on the contrary, the bank smoothes interest rate shocks, providing insurance against monetary policy shocks. The first result arises because the equilibrium quantity of loans is a decreasing and convex function of default costs. When negative shocks of real origin affect borrowers, increasing expected default costs, the bank reacts with a credit crunch, and by raising interest rates on loans. Consequently, in this case banking intermediation may contribute to the amplitude of business cycles, generating a financial accelerator. Strict regulatory requirements concerning the write-off of bad loans, however, are crucial for the efficiency of the banking industry, and for the health of the economic system as a whole. If the the bank is allowed to roll over loans that should be written off, the constraint on the size of the portfolio is virtually removed, since default costs can be indefinitely postponed as long as borrowers can pay back the interest. Almost any investment could in this case be financed, producing large distortions in the productive structure.

If the impact of a real shock affects different regions asymmetrically, banks operating in different regions face different variations of expected default costs. This causes the optimal size of the portfolios of the different banks to change differently. In the regions that suffer more, banks need to shrink the portfolio to a larger extent, reducing the issuance of loans and thus lending at higher rates. This mechanism amplifies the asymmetric impact of the shocks. For this reason, large interregional or international banks may be very important to reduce the asymmetric impact of real shocks.

Positive interest rate shocks induce the bank to increase the size of the portfolio by issuing more loans, because they become more profitable. The interest rate on loans set by the bank, thus, changes less than proportionally as market interest rates vary. Furthermore, the bank is more willing to provide deposit services when market interest rates are high. The desired optimal level of deposits depends, in fact, on the returns of the asset side of bank balance sheets. This implies that higher interest rates, which raise the returns of assets, induce a larger desired level of deposits. This supply side effect is normally neglected in monetary theory, but can explain the limited interest rate elasticity of the demand for money that is found in empirical studies. Even the instability of monetary aggregates may largely depend on the links between banks' assets and liabilities, as, in many circumstances, central banks need to control the issuance of loans in order to keep control of the growth of monetary aggregates. Conversely, according to the model, very low market interest rates push the bank to increasingly rely on market sources of finance. As market rates decline, market finance becomes increasingly competitive, since to provide deposit services the bank incurs industrial costs. The bank of the model reacts to lower interest rates, initially by reducing the buffer represented by interbank lending or short term securities, to become a net lender as the decline goes on. Long periods of low short-term market rates thus produce a strong incentive to finance lending by means of interbank borrowing or issuing paper, generating a substantial systemic liquidity risk.

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