Constrained Sequential Job Search

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July 30, 2013

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Abstract

We look at the optimal sequential job search strategy of an individual who faces a threshold wage constraint. We consider a model with two inter-linked frictional markets where obtaining a threshold wage in the labour market is a pre-condition for entry in a second market - here, marriage. We fully characterise the reservation wage function and point out an interesting (and robust) non-monotonicity property.

1 Introduction

The standard sequential job search problem looks at the optimal reservation wage under the assumption that accepted wages have no impact outside the labour market.[1] Sometimes however, job search in a frictional labour market may be affected by wage-related constraints outside this market. In some cases, obtaining a threshold wage may be a pre-condition for entry into a second - possibly frictional - market. For example, single individuals may need to find good enough jobs before they can consider finding a partner and getting married. In some other cases, the threshold wage may simply be a constraint imposed outside the labour market. One can think of mortgages that are conditional on various income levels.

In this paper we ask what is the effect of such constraints on the optimal sequential job search behaviour. In order to fully characterise the reservation wage strategy and establish the robustness of its properties, we first consider an example with two inter-linked frictional markets (labour and marriage). Then we discuss what happens if frictions in the second market vanish or if the utility from entry in the second market is large.

2 Setup

The economy consists of three types of agents: women, men and firms, all risk neutral. Time is continuous and agents discount the future at common discount rate r.

Men enter the market unemployed and single and use costless random sequential search to locate firms. Contact with a firm occurs at rate λ_0 and the (exogenous) distribution of wages offered is denoted by $F(w)$, continuous on the interval $[\underline{w}, \bar{w}]$. When employed at wage w and single, a man has flow payoff w.

Single employed men also look for potential partners, and know the extent to which women are selective about whom they marry. In particular, assume that women only marry men who earn wages at least as high as $T > 0$ (the threshold wage), and that they do not marry unemployed.[2] A single employed man meets a single woman at rate λ. A married man earning wage w enjoys flow payoff $w + y$, where $y > 0$ captures the non-material utility of matrimony (love). There is no on-the-job search and both women and men are faithful (no on-the-marriage search), so no divorce either. Singles and couples alike leave the economy at an exogenous rate δ.

[1] McCall (1970), Mortensen (1986)

[2] An assumption for now, shown to be true in equilibrium.

3 The reservation wage function

Overall, a man can be in one of three states: unemployed, employed at wage w and single (S), or earning a wage w and married (M). Denote his value of being unemployed by U and let $V_S(w)$ describe the value of being single and earning a wage w. Then, usual arguments imply that the Bellman equation solves

$$(r+\delta)U = \lambda_0 \int_{\underline{w}}^{\bar{w}} \max\left[V_S(w) - U, 0\right] dF(w)$$

Sequential search and the fact that utilities are increasing in wages imply that the optimal strategy for an unemployed has the reservation wage property: men only accept jobs with wage $w \geq R$. They choose R in the knowledge that women require T in the marriage market, rejecting men who earn wage $w < T$. Anticipating that $V_S(w)$ is not a continuous function (see below), the reservation wage R can thus be defined as

$$R = min\{w : V_S(w) \geq U\}$$

With no divorce, the value of being married and earning a wage w is $V_M(w) = \frac{w+y}{r+\delta}$.

Given T, we have

$$V_S(w) = \left\{ \begin{array}{lcc} \frac{w}{r+\delta} & \text{if} & w < T \\ \frac{w}{r+\delta} + \frac{\lambda}{(r+\delta+\lambda)(r+\delta)}y & \text{if} & w \geq T \end{array} \right\}. \tag{1}$$

Let $\underline{R}$ denote the reservation wage of men in a pure labour market equilibrium when the marriage market is irrelevant from the point of view of men. $\underline{R}$ is defined as the (unique) solution to

$$\underline{R} = \frac{\lambda_0}{r+\delta} \int_{\underline{R}}^{\overline{w}} \left[1 - F(w)\right] dw.$$

This is the lowest reservation wage R and it results under two scenarios. First, for $T < \underline{R}$, women marry anybody (T is very low) and hence men don't need to worry about their marriage prospects. In contrast, for $T > \overline{w}$, the threshold T is so high that women turn down everybody, and hence men are left to focus entirely on the labour market. Consequently, in both cases the optimal reservation wage is $\underline{R}$.

Define $\widehat{T}$ (derived below) as the highest threshold wage for which $R(T) = T$. Assume for now (to be confirmed later), that $\widehat{T} \in [\underline{R}, \overline{w})$. This implies the existence of two regions:

(a) For $T \in (\widehat{T}, \overline{w}]$, we have $R < T$.

In this region, men's reservation wage strategy implies $U = V_S(R) = \frac{R}{r+\delta} (< \frac{T}{r+\delta}$ here). For a given T, and using $V_S(w)$ from (1) we have

$$R = \lambda_0 \int_R^T \left[\frac{w}{r+\delta} - \frac{R}{r+\delta} \right] dF(w) + \lambda_0 \int_T^{\bar{w}} \left[\frac{w}{r+\delta} + \frac{\lambda}{(r+\delta+\lambda)(r+\delta)} y - \frac{R}{r+\delta} \right] dF(w),$$

or

$$R = \frac{\lambda_0}{r+\delta} \int_R^{\bar{w}} [w - R]\, dF(w) + \frac{\lambda_0}{(r+\delta)} \int_T^{\bar{w}} \frac{\lambda y}{(r+\delta+\lambda)} dF(w).$$

The first part on the right hand side is the standard expected wage gain from continued search. The second part captures the additional expected utility gain from having a wage that allows a man to get married.

Integration by parts of the first integral and simplification of the second imply

$$R = \frac{\lambda_0}{r+\delta} \left[\int_R^{\bar{w}} [1 - F(w)]\, dw + \frac{\lambda [1 - F(T)]}{(r+\delta+\lambda)} y \right]. \tag{2}$$

Again, the first term on the right hand side is standard: the reservation wage must compensate for the loss of the option of continued search for better wages. The second term relates to the man's marriage option. The possibility of contacting a firm (at rate λ_0) that offers a marriageable wage $w > T$ (which happens with probability $1-F(T)$), and subsequently meeting a woman (at rate λ) would leave him enjoying flow value y. If a worker were to accept a wage $w < T$, he would be giving up the option of happiness through marriage. Consequently, the reservation wage must compensate for this loss.

Please note that

$$\frac{\partial R(T)}{\partial T} = \frac{\lambda_0 \lambda \frac{\partial F(T)}{\partial T} y}{-(r+\delta+\lambda)(r+\delta+\lambda_0) + F(R)\lambda_0(r+\delta+\lambda)} < 0$$

and

$$\frac{\partial^2 R(T)}{\partial T^2} = \frac{\frac{\partial^2 F(T)}{\partial T^2} \left[r+\delta+\lambda_0(1-F(R))\right]^2 (r+\delta+\lambda)\lambda_0 \lambda y + \lambda_0^2 \lambda \left[\frac{\partial F(T)}{\partial T}\right]^2 \frac{\partial F(R)}{\partial R} y}{(r+\delta+\lambda)^2 \left[r+\delta+\lambda_0(1-F(R))\right]^3} \lesseqgtr 0$$

Using (2), $\widehat{T}$ as defined above is the solution to

$$\widehat{T} = \frac{\lambda_0}{r+\delta}\left[\int_{\widehat{T}}^{\overline{w}} [1-F(w)] + \frac{\lambda\left[1-F(\widehat{T})\right]}{r+\delta+\lambda} y\right] \tag{3}$$

From here, $\widehat{R} = \widehat{T}$ is the reservation wage that matches T so that all employed men are eligible to marry once they find a woman. Clearly, $\widehat{R} > \underline{R}$.

(b) For $T \in [\underline{R}, \widehat{T}]$, we have $R = T$.

For T in this region, a reservation wage $R < T$ (and therefore with $U < \frac{T}{r+\delta}$) is no longer optimal. We need to consider a reservation wage $R = T$, where $U \geq \frac{T}{r+\delta}$. Now, it must be the case that $U < V_S(T)$ because R is not obtained from $U = V_S(R)$. Therefore, men will use a reservation wage $R = T$ if and only if

$$\frac{T}{r+\delta} \leq U \leq \frac{T}{r+\delta} + \frac{\lambda y\left[1-F(T)\right]}{(r+\delta)(r+\delta+\lambda)} = V_S(T)$$

In principle, one could also consider $R > T$, but this is not optimal: if R was higher than T, that would mean $R = \underline{R}$, as men would ignore the marriage market. However, this would be a contradiction for $T > \underline{R}$.

Throughout, we assumed that women don't marry unemployed men. If they did - since there is no divorce, a man's reservation wage would drop to $\underline{R}$. Thus, marrying an unemployed is inconsistent with only accepting men earning $w > T > \underline{R}$.

The above provides a full characterisation of the optimal search strategy in the presence of a wage-related constraint T. The reservation wage function exhibits an interesting non-monotonicity property, described and explained as follows:

Starting from $T = \underline{R}$, as women become more selective in the marriage market (T increases), men also become more selective in the labour market: they need to earn a high wage in order to become attractive marriage partners, and will not accept anything less. In this range, given a relatively low threshold wage, any wage $w < T$ not only precludes marriage but is also too low - men will reject it.

The feedback, however, is non-monotonic. For higher values of T, the relationship changes. At some point ($\widehat{T}$), a wage $w < T$ can still be high enough so that men accept it even if it precludes marriage. From here on, as women become even more selective, males tend to gradually give up on the marriage market. This is simply because they find it increasingly difficult to encounter wages that attract women. As a consequence, the reservation wages in the labour market gradually fall, to the point where men give up completely.

The above results have been obtained within a framework where both markets are frictional, and for a given y. How robust (general) is this non-monotonicity property? First, is it dependent on the existence of frictions in the second market? Second, recall that the flow value y constitutes the link between the two markets which ensures that the job searchers would in principle like to gain entry into the second market. Can $R < T$ still be part of the optimal reservation wage strategy if this utility is very large?

To answer the first question, note that for an unemployed man the discounted expected utility of marriage is $\frac{\lambda_0 \lambda[1-F(T)]}{(r+\delta+\lambda)} y$. It is easy to see that, even if λ tends to ∞, the value attached to the option of marriage is bounded as long as $y < \infty$. This is because, before even contemplating marriage, an unemployed must find a job first.[3] What this means is that the wage-related constraint will always influence job search behaviour, with or without frictions in the second market.

Furthermore, recall that $\widehat{T}(y)$ is

$$\widehat{T} = \frac{\lambda_0}{r+\delta} \left[\int_{\widehat{T}}^{\bar{w}} [1 - F(w)]\, dw + \frac{\lambda \left[1 - F(\widehat{T})\right]}{(r+\delta+\lambda)} y \right].$$

It is straightforward to see that for $y = 0$, this gives $\widehat{T} = \underline{R}$. We can rewrite the above as

$$y = \frac{(r+\delta+\lambda)\left[\widehat{T}(r+\delta) - \lambda_0 \int_{\widehat{T}}^{\bar{w}} [1 - F(w)] dw\right]}{\lambda_0 \lambda \left[-1 + F(\widehat{T})\right]},$$

so $\underset{\widehat{T} \to \overline{w}}{lim}\, y = \infty$ (as the limit of the numerator is a positive constant, while the limit of the denominator is zero). Since $\widehat{T}$ is an invertible function, it follows that $\underset{y \to \infty}{lim}\, \widehat{T} = \bar{w}$. The range with $R < T$ exists as long as y is bounded.

[3] Conversely, this value is infinite for $y = \infty$, even if $\lambda < \infty$.

Finally,

$$\frac{\partial \widehat{T}}{\partial y} = \frac{\lambda_0 \lambda \left[1 - F(\widehat{T})\right]}{(r+\delta+\lambda)\left\{r+\delta+\lambda_0\left[1-F(\widehat{T})\right]\right\} + \lambda_0 \lambda \frac{dF(\widehat{T})}{d\widehat{T}} y} > 0,$$

which is positive since $\frac{\partial F(\widehat{T})}{\partial \widehat{T}} > 0$.

The findings are summarised in the following Proposition:

Proposition 1 *For any y bounded, we have $\widehat{T} \in (\underline{R}, \overline{w})$ with $\frac{\partial \widehat{T}}{\partial y} > 0$ and*
(i) $R = \underline{R}$ for $T \leq \underline{R}$ and $T \geq \overline{w}$;
(ii) $R = T$ for $T \in (\underline{R}, \widehat{T}]$;
(iii) $R < T$ for $T \in (\widehat{T}, \overline{w})$ with $\frac{\partial R}{\partial T} < 0$ and $\frac{\partial R}{\partial y} > 0$.

Observe that although $\frac{\partial R}{\partial y} = 0$ in (ii), the marriage market nevertheless affects the labour market (as $R = T$). Furthermore, please note that the wage distribution $F(w)$ impacts on the reservation wage only in the $R < T$ section.

Figure 1 below illustrates our results.[4]

4 Discussion

We have investigated the effect of a wage-related constraint in one market (e.g. marriage) on the optimal job search strategy in the labour market. We have shown that the reservation wage function is non-monotonic. Intuitively, this is because with two inter-linked markets, wages play a dual role: they provide income from employment *and* allow (or not) access to the second market. If the constraint is relatively low, the reservation wage will match it; if the constraint is high, a relatively high wage could now be acceptable even if it precludes entry to the other market.

There will always be a range of threshold wages (T) for which the optimal reservation wage R is lower than T. When this is the case, the reservation wage will decrease for T higher and higher, because job searchers will find it harder and harder to encounter high enough wages, so they gradually give

[4] For a given y and with F uniform.

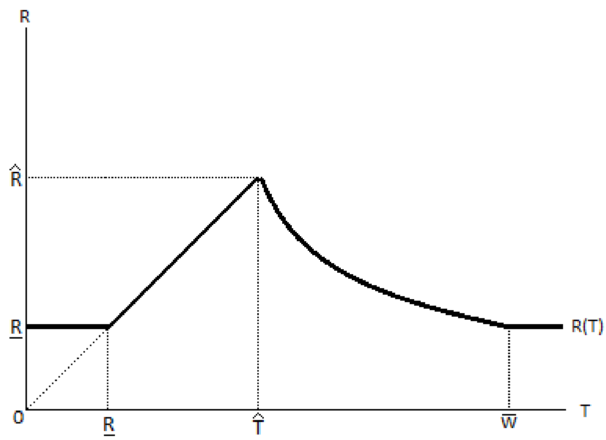

Figure 1

up on the second market. Of course, the rate at which this happens depends on the actual shape of the wage distribution. Indeed, recall that $\frac{\partial^2 R(T)}{\partial T^2} \lesseqgtr 0$. For convenience, we illustrated the case with an uniform distribution (so the $R < T$ range was convex). Alternatively, with F normal, the $R < T$ range starts off being concave then turns convex. Furthermore, if the threshold wage T was not unique (possibly distributed over a continuum of values), that would also affect the rate at which R decreases with T.

The non-monotonicity property of the reservation wage function is robust in two ways. First, it does not rely on the existence of frictions in the second market - all that matters is that there is a wage-related constraint linked to this market (indeed, the threshold wage itself could be either exogenous or endogenous). Secondly, the non-monotonicity property holds as long as the utility from entry to the second market is bounded.

As a consequence, the reservation wage function of a constrained sequential job search problem can be applied to various models that explore the interaction between a frictional market and an other (possibly frictional) market.[5]

[5] Bonilla and Kiraly (2013) consider frictional labour and marriage markets with an endogenous threshold wage and show that the widely reported marriage wage premium can be an equilibrium outcome purely as a result of the non-monotonic nature of the reservation wage function.

References

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