Search, Work and Marriage

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Abstract

I analyse an economy where a search labour market and a matching marriage market interact. The economy is populated by homogeneous workers, firms and marriage partners (MPs). Workers simultaneously search for firms in order to work and for MPs in order to marry. Firms post wages to attract workers. MPs look for workers in order to marry. I assume that married workers receive a pre-determined flow utility, and married MPs derive flow utility equal to the worker's earnings. This provides the link between the markets. Noisy search in the labour market generates a distribution of wages. I show that the so called married wage premium can be the consequence of frictions in both markets, without having to resort to the typical explanations. In one equilibrium, MPs marry only high earners, while workers accept wages that render them "unmarriageable". The workers' reservation wage must compensate them for the loss of marriageability in addition to the option of continued search for better wages. This affects the distributions of wages offered and earned, which are in turn crucial in the MPs decision to marry/reject low earners. In another equilibrium, worker's always find it optimal to increase their reservation wage in just the amount required to become marriageable, generating instead a one way relationship from MPs behaviour to worker's reservation wage

Equilibria in a model with a search labour market and a matching marriage market.

This paper analyses the equilibria in an economy where a search labour market and a matching marriage market interact. The economy is populated by ex-ante homogeneous workers, ex-ante homogenous firms, and ex-ante homogeneous marriage partners (MPs). Workers simultaneously search for firms in order to work and for marriage partners in order to marry. Firms post wages to attract workers; while *MPs* look for workers in order to marry. I assume that married workers receive a pre-determined flow utility; and that married MPs derive flow utility equal to the worker's earnings (be it wage or unemployment benefit). This provides the link between the two markets. I use noisy search in the labour market to generate a distribution of wages offered and of wages earned[1]. In this set-up, a worker's search for a firm is analogous to a marriage partner's search for a worker, and both will use reservation wage strategies in their search efforts[2]. The decisions on reservation wages are interdependent: workers decide on their own reservation wage taking as given the marriage partners' reservation wage and the shape of the wage offer distribution. Marriage partners decide on their own reservation wage taking as given the worker's reservation wage and the shape of the distribution of wages earned.

To my knowledge, there is no other paper that analyses equilibrium in a model with two interacting frictional markets where relationships in both markets are long-term and interdependent decisions are taken by all sides of the market. I believe this to be the main theoretical contribution of this paper. Burdett, Lagos and Wright (2003,2004) present models in which workers in a frictional labour market encounter opportunities to commit a crime at a less that infinite rate, which is eventually endogenised. The workers decide on the reservation wage and on what they call the "crime" wage: workers will not commit a crime if earning more than that. A big difference is that in Burdett, Lagos and Wright (2003), it is the workers who make all the decisions, while in this paper all agents make interdependent decisions[3].

[1] The modeling of noisy search is based on Burdett and Judd (1983). I assume that when workers contact firms, they may have contacted one or two firms, with given probability strictly between 0 and 1. When firms are contacted by workers, they do not know if the worker has contacted one or two firms. This gives rise to equilibrium wage dispersion as firms balance the higher probability of a hire when offering higher wages with the lower profit given a hire is made.

[2] A marriage partner may not be willing to marry anybody earning less than a given wage.

[3] In Burdett, Lagos and Wright (2004), firms post wages, but it is still true that workers

There is wide empirical evidence (and more than wide anecdotal evidence) to support the idea that labour market performance of the prospective partner is considered when making decisions about marriage. Ginther and Zavodny (2001) find evidence that men are selected into marriage on the basis of their higher earning capabilities. They compare the wage premium among men whose marriage was triggered by a pregnancy and was therefore followed by a birth within seven months; with those whose marriage was not followed by birth. Ginther and Zavodny (2001) find that "married men with a pre-marital conception generally have a lower return to marriage than other men do"[4]. Gould and Paserman (2003) argue that 25% of marriage rate decline since the 1980s can be explained by the increase in male wage inequality. The argument is that wage inequality increases the option value for women to search longer for a husband[5]. Loughran (2002) models women's search for marriageable men in a similar manner as in this paper, but Loughran (2002) is a decision theory model[6], not an equilibrium model. Hence, he completely ignores the role played by workers (searching for a job) and by firms (posting wages), and of course the equilibrium consequences.

Lundberg (2005) makes a call for research into the interdependence of decisions about work and marriage. The model presented here attempts to be a theoretical contribution to one of the dimensions of such interdependence and its consequences. In particular, the paper shows that the so called "married wage premium" (or, more general, a correlation between men's wages and marital status) can in some circumstances be the equilibrium consequence of search frictions in the two markets. This is completely unrelated to the traditional explanations for a link between wages and mar-

deicde on both the reservation wage and the crime wage; while in here workers must take as given the reservation wage used by MPs.

[4] For evidence on this, see Zavodny (1998) and Marsiglio (1987).

[5] The empirical literature on the married wage premiume is extensive and mixed. It is not my intention to provide a review of it here.

[6] Which is then used to motivate the non structural estimation of the relationship between wage distribution of males and age at first marriage of females.

ital status based on specialization in the labour market and on unobserved characteristics that are valuable both in the labour the marriage market [7]. Regarding the former, specialisation in the labour market after marriage is not introduced at all in the paper. Regarding the latter, this would require some kind of heterogeneity, so that some agents have the said unobserved characteristics, while some do not.

I know of no other paper that analyses the equilibrium interaction of two frictional markets, both giving rise to long term relationships. The labour market and the housing market provide another example of inter-related frictional markets with long term relationships. There is no reason why the methodology used here could not be adapted to study the interaction of those two markets. This is a line for further related research.

The details of this paper are set out in Section 1 below. I show that three pure strategy equilibria exist in the environment generally described so far, complemented by one mixed strategy equilibrium. In the simplest equilibrium, which I term the Smitten Equilibrium, MPs accept all workers for marriage, regardless of their employment status or current wage if employed. In this case, the distributions of wages offered and earned are so compressed that it does not make sense for MPs to reject marriage to any worker in order to engage in further search. In what I term the Very Picky (VP) Equilibrium, MPs reject marriage to unemployed workers and to low earners, and only marry employed, high earning workers. . In what I term the Picky Equilibrium (P), MPs reject marriage to unemployed workers, but accept marriage to all employed workers, regardless of the wage earned.

In the VP equilibrium, the utility workers derive from marriage is particularly relevant. It affects workers' reservation wage, since the reservation wage must compensate workers for the loss of marriageability in addition to the option of continued search for better wages. This affects the distributions of wages offered and of wages earned, which in turn are crucial in the

[7] See Becker (1991) and Nakosteen and Zimmer (1987) respectively.

MPs decision to accept low earners for marriage or not. As a result, there is a two way equilibrium relationship between workers reservation wage and MPs behaviour. Further, there is an endogenous correlation between wages and marital status. In the P equilibrium, the worker's problem has a corner solution: they always find it optimal to increase their reservation wage in just the amount required to become marriageable. In this case, there is a one way relationship, as the MPs reservation wage affects the workers reservation wage; and there is no correlation between wages and marital status.

I use some simplifying assumptions, the removal of which is the basis for further or ongoing research. In particular, I assume that divorce is infinitely costly and therefore never occurs. When agents accept marriage, they do so knowing that they will never get divorced. This allows me to solve the problem analytically and to obtain interesting results about the marriage problem. In the conclusion, I discuss the consequences of allowing for divorce and preliminary results of ongoing research. A further assumption is that single MPs enjoy a predetermined flow utility, which I call X. In this set-up, X can have many interpretations, like the value of living with parents, the value of accessing a low skill competitive labour market, or the possibility of marrying differently skilled workers. I discuss in the conclusion consequences of modeling X in more detail and preliminary results.

Section 1 below sets up the model and the strategies for the firms, the workers and the marriage partners. Sections 2 to 4 present the pure strategy equilibria briefly described above taking arrival rates as parametric. Section 5 endogenises the arrival rates and separates the parameter space into the three pure strategy equilibria described above. Section 6 presents a mixed strategy equilibrium and Section 7 concludes.

1 The Model.

In this section I set up the model and the assumptions relevant to the three types of agents: Firms, Workers and Marriage Partners.

Firms Individul firms post wages and contact workers who are either single or married. Firms can wage discriminate according to the workers' marital status. Consider the problem vis-a-vis single workers first. Each firm takes as given the reservation wage of unemployed-single workers (R) and the distribution of wages for single workers offered in the market $G(w)$. The same is true about unemployed-married workers whose reservation wage I denote R_m, and the distribution of wages for married workers is $I(w)$ When an individual firm contacts a worker, the worker may have contacted only her with probability g or one other firm with probability $1 - g$, where $0 < g < 1$.. If a firm offers wage w, and worker accepts, flow productivity is p. The match destroys if the worker dies, at an exogenous rate δ. I model the labour market as a pure search market, hence firms can absorb all workers that contact her and accept her wage offer.

Workers Workers take as given the reservation wage of MPs (T)[8], and the distribution of wages offered ($G(w)$ for singles and $I(w)$ for marrieds). Unemployed workers decide their reservation wage (singles decide on R and marrieds decide on R_m). When workers make a contact, with probability g they contact only one firm and with probability 1-g they contact two firms. Hence, the distribution of wages faced by single or married workers in their search effort is different from that respective distribution of wages offered. I denote by $F(w)$ the distribution of wages faced by single workers and by $H(w)$ the distribution faced by married workers. All workers, regardless of their marital status, receive unemployment benefit b while unemployed. When workers are married, they enjoy flow value m, regardless of their labour market status (in addition to their wage if they are employed, and to the unemployment benefit if they are unemployed). Workers contact firms

[8] This means marriage partners will not marry a worker earning $w < T$.

at rate λ_0 when single and there is no on- the-job search. They contact MPs at rate λ_m when single. They die at rate δ whatever their employment status.

Marriage Partners (MPs). MPs take as given the distribution of wages earned (by single workers, who got their job while single), $G(w)$[9] (including the reservation wage R); they decide on their own reservation wage, T, *i.e.*, they will not marry anybody earning less than T. When they are married to an employed worker earning w, they enjoy flow value w. When they are married to an unemployed worker receiving unemployment benefit b, they enjoy flow value b[10]. *MPs* contact workers at rate η and die at rate δ when single. I assume that married MPs die if their partner dies, and this happens at rate δ. Notice that the marriage market is a matching market with non-transferable utility.

In the model, time is continuous and I only analyse steady state equilibria.

Wage Distributions

From Section 2, I characterise the possible equilibria in this model. Here I derive results about the wage distributions that are valide for all those equilibria.

Equal profits condition and distributions of wages offered: $G(w)$ and $I(w)$. Assume a firm offers wage $w \geq R$ and consider the firm's problem vis-a-vis single workers. Given a worker accepts the job offer, the firms discounted profits from employing that worker are $\pi(w) = \frac{p-w}{r+\delta}$. Given that a worker has been contacted, and wage $w \geq R$ is offered, the expected profits are

$$\Pi(w) = (1-g)G(w)\pi(w) + g\pi(w).$$

Denote the lowest and highest wages offered by $\underline{w}$ and $\bar{w}$ respectively. For any wage w such that $\underline{w} \leq w \leq \bar{w}$, in equilibrium it must be true that $\Pi(\underline{w}) =$

[9] Since there is no on the job search, the distribution of wages offered is the same as the distribution of wages earned.

[10] *MP*s do not participate in the labour market. It is not an unrealistic assumption to think of agents that do not engage in the labour market. Even today, this is the case for women in most developing countries.

$\Pi(w)$. Assuming $R = \underline{w}$ (which below I argue to be true in equilibrium, for the standard arguments), then $\Pi(\underline{w}) = g\pi(w)$ since $G(\underline{w}) = 0$. Then:

$$\begin{aligned}
\Pi(\underline{w}) &= \Pi(w) \Rightarrow g\pi(\underline{w}) = (1-g)G(w)\pi(w) + g\pi(w) \\
G(w) &= \frac{1-g}{g}\frac{w-\underline{w}}{p-w}, \\
G(\bar{w}) &= 1 \Rightarrow \bar{w} = gp + (1-g)\underline{w}
\end{aligned}$$

Notice, $G(w)$ is continuous along its support.. $R = \underline{w}$ is therefore true, for the standard reasons: *i)* A wage $w < R$ will not be offered by any firm because no worker will accept it. *ii)* Assume $\underline{w} > R$, then $F(\underline{w}) = F(R) = 0$. Then any firm offering $\underline{w}$ can reduce its wage offer all the way to R and increase its expected profits.

The problem vis-a-vis married workers is analogue to the above, with the difference that the minimum and maximum wages need not be the same as for single workers. Denote by $\underline{w}_m$ and $\bar{w}_m$ the minimum and maximum wage respectively in the wage distribution offered to married workers. Then it is given (analogously) by $I(w)$ where:

$$I(w) = \frac{1-g}{g}\frac{w-\underline{w}_m}{p-w}, \bar{w}_m = gp + (1-g)\underline{w}_m$$

where $\underline{w}_m = R_m$[11].

Search-relevant wage distributions: $F(W)$ and $H(w)$. Givent that workers contact one firm or two firms with the respective probabilities g and $1-g$, the distribution of wages faced by single [married] workers in their search effort is given by:

$$\begin{aligned}
F(w) &= gG(w) + (1-g)G(w)^2 \Rightarrow F(w) = \frac{(1-g)^2}{g}\frac{(w-\underline{w})(p-\underline{w})}{(p-w)^2} \\
H(w) &= gI(w) + (1-g)I(w)^2 \Rightarrow H(w) = \frac{(1-g)^2}{g}\frac{(w-\underline{w}_m)(p-\underline{w}_m)}{(p-w)^2}
\end{aligned}$$

[11] For reasons analogue to those exposed in footnote 8.

Below I study the possible equilibria in this model. I term them the Very Picky Equilibrium (VP)[12], the Picky Equilibrium (P)[13], and the Smitten (S) Equilibrium[14] . In the sections below, when the subscripts vp, p or s appear on a variable, this denotes that the variable takes the value corresponding to the VP, P or S equilibrium respectively.

2 The Very Picky Equilibrium (VP).

In the VP equilibrium, MPs reject marriage to some low-wage employed workers and with unemployed workers[15]. This means that, in equilibrium, unemployed workers are willing to accept wages that make them unmarriageable. I first analyse the worker's problem and then go on to analyse the MP's problem. Then I show when the equilibrium obtains.

Workers. Assume single workers decide on a reservation wage $R = R_w$. Then, following the desired properties of the VP equilibrium, I require that

$$i) R_w < T < \bar{w}, \quad ii) R_w > b$$

Condition $i)$ ensures that unemployed workers are willing to accept wages that make them unmarriageable. Condition $ii)$ ensures that the minimum wage accepted by unemployed workers is strictly higher than their unemployment benefit b[16]. If working at a wage $x < T$, the worker's payoff is

[12] Becaue MPs reservation wage T is higher than the workers reservation wage R, which means that some employed workers, those earning w such that $R < w < T$, are unmarriageable.

[13] Becaue MPs reject marriage to unemployed workers but accept all employed workers.

[14] Becaue MPs marry all workers disregarding their labour market status.

[15] This is not necesarilly true always. There may be reasons why a MP could prefer marriage to an unemployed worker over marriage to a low earner, but these are not built into this model. Uncertainty over the unemployed workers productiviy is an example, as this would imply uncertainty over the workers expected performance in the labour market.

[16] $R_w < b$ is not rational from the unemployed worker's point of view, and $R = b$ would lead to a qualitatively different type of equilibrium as will become clear later

given by $V_1(x)$ defined by $rV_1(x) = x - \delta V_1(x)$, since there is no expectation of marrying. If working at a wage $x \geq T$, the worker's payoff is given by $V_2(x)$ where $rV_2(x) = x + \lambda_m(V_3(x) - V_2(x)) - \delta V_2(x)$ (since λ_m is the rate at which marriageable workers meet MPs), where $V_3(x)$ is the payoff of being married and working at wage x. If working at a wage x and married, the workers payoff is given by $V_3(X)$, where $rV_3(x) = w + m - \delta V_3(x)$. The payoff of being single is given by

$$rV_0 = b + \lambda_0 \int_{\underline{w}}^{T} [\max(V_1(x), V_0) - V_0)]\, f(x)dx + \lambda_0 \int_{T}^{\bar{w}} [\max(V_2(x), V_0) - V_0]\, f(x)dx - \delta V_0 \tag{1}$$

In equation (1), a worker faces a wage offer distribution $F(w)$. He receives b while unemployed. He contacts firms at rate λ_0. If the contacted firm offers a wage x such that $R_w < x \leq T$, then he must choose between accepting the job which makes him unmarriageable with payoff $V_1(x)$ or remaining single. If the firm offers a wage x such that $T \leq x < \bar{w}$ then the worker must choose between accepting the job which makes him marriageable with payoff $V_2(x)$ or remaining single. The worker dies at rate δ. Given a wage offer w has been received by a worker, $\frac{\partial V_1(w)}{\partial w} > 0$ and $\frac{\partial V_0}{\partial w} = 0$. Then, the standard definition of a reservation wage implies $V_1(R_w) = V_0$, $w \gtreqless R_w$ $\Rightarrow V_1(w) \gtreqless V_0$. Hence, the worker accepts any wage $w \geq R_w$, and (1) can be written[17]:

$$rV_0 = b + \lambda_0 \int_{R_w}^{T} [V_1(x) - V_0)]\, f(x)dx + \lambda_0 \int_{T}^{\bar{w}} [V_2(x) - V_0]\, f(x)dx - \delta V_0 \tag{2}$$

Integration by parts of (2) using $V_1(x), V_2(x), V_3(x)$ and $V_1(R_w) = V_0$ yields

$$R_w = b + \frac{\lambda_0 \lambda_m m(1 - F(T))}{(r + \partial + \lambda_m)(r + \delta)} + \lambda_0 \int_{R_w}^{\bar{w}} \frac{1 - F(x)}{r + \delta} dx$$

[17] Considering as well that $V_2(w) > V_1(w)$

In the above equation, the first and third elements of the right hand side are standard: the reservation wage must compensate the worker for the loss of unemployment benefit and for the option of continued search for better wages. The second term relates to the marriage option. If the workers accept wages that make them unmarriageable, they are giving up the expected utility attached to marriageability[18]. The reservation wages must compensate them for this loss. In the limit as $r \to 0$, and using $F(w)$ as in Section 1 with $R_w =\underline{\mathrm{w}}$ and $\bar{\mathrm{w}}= \frac{p+\mathrm{w}}{2}$, this yields

$$R_w = b + \frac{k_0 k_m m \left[1 - \frac{(R_w - T)(-p+R_w)}{2(p-T)^2}\right]}{1+k_m} - k_0 \Lambda(-p+R_w) \tag{3}$$

where $\Lambda = \frac{g(2g-1)+\ln(\frac{1}{1-g})(1-g)^2}{g} > 0$and $k_i = \frac{\lambda_i}{\delta}$.

From (3), it is possible to derive the necessary results to characterise the behaviour of R_w in the range $R_w < T < \bar{w}$. The closed form solution for R_w from (3) is rather cumbersome, and is therefore relegated to the Appendix. To avoid technical complications, I will assume $m < m_a$, where $m_a = \frac{(1+k_m)(p-b)g}{(g+1+k_0\Lambda)k_m k_0}$. The intuition behind this condition is easier to explain after stating and explaining Proposition 1 below. In Proposition 1, I evaluate R_w in the two extremes: when $T = R_w$ (as low as it can be) and when $T = \bar{w}$ (as high as it can be); and I characterise the behaviour of $R_w(T)$ in the region $R_w(T) < T < \bar{w}$. The main message of Proposition 1 states that $R_w(T)$ is a downward sloping concave curve. The intuition is provided after the Proposition.

Proposition 1.

[18] Notice this is just the flow value of marriage (m) discounted by the relevant factors. Upon continued search, flow utility m would be enjoyed if a firm is contacted (which happens at rate λ_0) that offers a marriageable wage (which happens with probability $1 - F(T)$ given a firm has been contacted); and then a marriage partner is contacted (which happens at rate λ_m) .

i) $T = R_w(T)$ implies $R_w(T) = R_1$ and $T = T_1$ where

$$R_1 = T_1 = \frac{(1+k_m)(b+k_0\Lambda p) + 2k_0k_mm}{(1+k_m)(1+k_0\Lambda)}$$

ii) $T = \bar{w}$ implies $R_w(T) = R_2$ and $T = T_2$ where

$$R_2 = \frac{b+k_0\Lambda p}{1+k_0\Lambda} < R_1, \quad T_2 = \frac{p(g+k_0\Lambda) + b(1-g)}{1+k_0\Lambda} > T_1$$

iii) $T_2 > T_1$ and (3) represents a downward sloping and concave curve in R_w, T space in the range $R_w < T < \bar{w}$.

Proof. See appendix.

Figure 1 exemplifies the situation. The intuition for Proposition 1 and Figure 1 is as follows:

i) If m is very high, marriage is too valuable for workers. They would *never* be willing to accept an unmarriageable wage, as that would mean giving up the prospect of enjoying m altogether.

ii) Assume m is high but not so high ($m < m_a$ satisfies "m is not so high"). Hence, workers could be willing to accept a non-marriageable wage under certain conditions. Assume as well that $R_w(T) = T$. As T goes up, workers have less incentive to reject any wage $x < T$, since further search is less likely to produce a marriageable wage. This implies $R_w(T)$ goes down.

iii) If m is very high, the effect of an increasing T on $R_w(T)$ is very high. Hence, as T goes up, $R_w(T)$ falls very fast. If $m \geq m_a$ as defined above, then $R_w(T)$ falls below b before $T = \bar{w}$. From then on, even as T continues increasing, equation (3) no longer describes the behaviour of R_w, as it would be irrational for workers to accept a lower reservation wage. I am avoiding this last complication by assuming $m < m_a$.

Marriage Partners. Assume MPs know workers use reservation wage $R = R_{mp}$. The properties of the VP equilibrium require

$$i) R_{mp} < T < \bar{w}, \quad ii) R_{mp} > b$$

MPs enjoy X while single[19], and they contact employed marriageable workers at rate η_{vp}. If they only accept marriage with employed workers earning $w \geq T > R_{mp}$ then their payoff is given by

$$rM_1 = X + \eta_{vp} \int_T^{\bar{w}} [M_2(x) - M_1]\, g(x)dx - \delta M_1 \tag{4}$$

In (4), given a contact with a single-employed worker earning $T < w < \bar{w}$, marriage occurs yielding payoff $M_2(x)$ (the payoff of an MP married to an employed worker earning wage x. The worker's wage is a random draw from $G(w)$[20]. The MP dies at rate δ. The value $M_2(x)$ is given by

$$rM_2(x) = x - \delta M_2(x) \tag{5}$$

In (5) above, if the MP is married to an employed worker, then its status will only change if death arrives, which happens at rate δ. Given a contact with a worker earning w, notice that $\frac{\delta M_2(w)}{\delta w} > 0$ and $\frac{\delta M_1}{\delta w} = 0$. Then $M_2(T) = M_1$, $M_2(T) \gtreqless M_1$ if $w \lesseqgtr T$. Integration by parts of (4) using (5) and evaluating in the limit as $r \to 0$ implies

$$T = X + \rho_{vp} \int_T^{\bar{w}} [1 - G(x)]\, dx$$

where $\rho_{vp} = \frac{\eta_{vp}}{\delta}$. Using $G(x)$ as above and integrating, the above can be written as

$$T = X + \rho_{vp} \left\{ \left[ln \left(\frac{(p - R_{mp})(1-g)}{p - T} \right) - 1 \right] (p - R_{mp}) \frac{(1-g)}{g} + \frac{p - T}{g} \right\} \tag{6}$$

I now characterise the behaviour of (6) when $b < R_{mp}(T) < T < \bar{w}$. Proposition 2 below lists all required information to sketch the graph of (6) in

[19] There are many possible interpretations for X: Living at home, working in a low wage competitive labour market, the possibility of marrying differently skilled workers, etc. A more detailed characterisation of X is discussed in the conclusion.

[20] The distribution of wages *earned* by single-employed workers.

$R_{mp}(T), T$ space . Such a graph is depicted in Figure 2. To state Proposition 2, I follow the same strategy used for Proposition 1: I evaluate $R_{mp}(T)$ in the extremes where $T = R_{mp}(T)$ and when $T = \bar{w}$, and I characterise $R_{mp}(T)$ when T satisfies $R_{mp}(T) < T < \bar{w}$. The main message of Proposition 2 is that $R_{mp}(T)$ is an upward sloping concave curve. The intuition is provided after the Proposition..

Proposition 2. If $p > X$, then

i) $T = R_{mp}(T)$ implies $T = T_3$, $R_{mp}(T) = R_3$, where

$$R_3 = T_3 = \frac{\rho_{vp} p \Phi + Xg}{g + \rho_{vp}\Phi} < \bar{w}, \Phi = \ln(1-g)(1-g) + g$$

ii) $T = \bar{w}$ implies $T = T_4$ and $R_{mp}(T) = R_4$, where

$$R_4 = \frac{X - gp}{1 - g} < \bar{w}, \quad T_4 = X$$

iii) $R_3 > R_4, T_3 > T_4$ and (6) represents an upward sloping and convex line in $R_{mp}(T), T$ space for $R_{mp}(T) < T < \bar{w}$.

Proof. See appendix.

Following the results in **Proposition 2**, one can sketch the graph of (6) as in Figure 2. The intuition for Proposition 2 and Figure 2 is as follows: As the worker's reservation wage (Rmp, which is taken as given by MPs) increases, the distribution of wages offered shifts up. This is because firms respond to worker's reservation wage, *i.e.* $G(w)$ is endogenous. Given a better distribution of wages earned, MP's become pickier in accepting marriage, since the option of continued search for a higher earner is more valuable. This describes the main message of Proposition 2: a positive relationship between MP's reservation wage (T), and worker's reservation wage, (R_{mp}).

An equilibrium exists if the functions $R_w(T)$ and $R_{mp}(T)$ cross while $R_w(T) < T < \bar{w}$ and $R_{mp}(T) < T < \bar{w}$. In order to state Lemma 1 below, I first define

$$X_a = \frac{p(g + k_0\Lambda) + (1-g)b}{1 + k_0\Lambda} < p$$

Lemma 1. $X = X_a$ implies $R_4 = R_2$ and $T_4 = T_2$. This implies a situation as depicted in Figure 3.

Proof. See appendix.

Lemma 2. As X decreases, $R_w(T)$ remains unchanged and $R_{mp}(T)$ shifts to the left. If $X = X_a - \epsilon$, $\epsilon > 0$ then the VP Equilibrium obtains. The situation is as depicted in Figure 4.

Proof. See appendix.

The intuition for Lemma 2 is as follows: As X decreases, MPs utility when single decreases. This implies that they give up less when accepting marriage to an MP, hence they become less picky in accepting marriage. This implies that, ceteris paribus, their reservation wage, T,decreases. Graphically, this means $Rmp(T)$ shifts to the left.

Lemma 3. $T_1 \geq T_3$ if and only if $X \leq X_b$, where

$$\begin{aligned} X_b &= \frac{2b\left[g + \Phi\rho_{vp}\right]}{(1 + k_0\Lambda)g} + \frac{k_0 k_m m\left[g + \Phi\rho_{vp}\right]}{g\,(1 + k_0\Lambda)\,(1 + k_m)} \\ &\quad + \frac{(k_0\Lambda g - \rho_{vp}\Phi)p}{(1 + k_0\Lambda)g} \end{aligned}$$

and $m < m_a \Rightarrow X_b < X_a$. In this case, the situation is as depicted in Figure 5, and the VP equilibrium does not obtain if this is the case: X is so low that $Rmp(T)$ has shifted too much to the left.

Proof. See appendix.

Proposition 3. The VP Equilibrium obtains if $X_b < X < X_a$.

*Proof. See appendix*Following Lemmas 1-3 and by inspection of the associated Figures, if the condition in Proposition 3 hold, the situation is as depicted in Figure 4.

Notice,

i) When the VP equilibrium obtains, there is a correlation between wages and marital status. Married workers are necessarily employed at a wage $w > T$, while single workers can be unemployed or employed at a wage $w > R$, where $R < T$.

ii) An increase in X (that does not take X above X_a) will make MPs more picky and they will increase their reservation wage T (as stated in Proposition 3). As a result, workers find it optimal to reduce their reservation wage, as they now have a smaller incentive to wait for (less probable) marriageable wages. Hence, there is a negative relationship between X and $R:$ As the MPs flow utility when single increases, they will increse their reservation wage T, causing a decrease in workers reservation wage R. As a consequece, the endogenous link between wages and marital status becomes stronger.

iii) As the value workers give to marriage increases (an increase in m that keeps $X_b < X < X_a$) this will increase their reservation wage R, as it must compenstae them for a bigger loss since marriageability is now more valuable. It is easy to show formally that this occurs through an upshift of R_w and this causes an increse in the equilibrium T as well.

3 The Picky Equilibrium (P).

In the P equilibrium, MPs marry workers if and only if these are employed. I construct this equilibrium by proposing that all workers and MPs use the same reservation wage, so $T = R = T_p = \frac{\rho_{vp} p\Phi + Xg}{g+\rho_p \Phi}$. If this is true in equilibrium, then it implies $\underline{\text{w}}= T_p$. The proof of Proposition 5 below shows when no individual worker or MP has an incentive to deviate from this strategy. The problem for an MP who is single is therefore to choose a reservation wage T, assuming that the minimum wage in the distribution of earned wages[21] is given by T_p.

$$rM_{1,\underline{\text{w}}=T_p} = X + \eta_p \int\limits_{T}^{\bar{w}} \left[M_2(x) - M_{1,\underline{\text{w}}=T_p}\right] g(x)dx - \delta M_1 \qquad (7)$$

where $G(w)$ is as given in Section 1 but using $\underline{\text{w}}= T_p$. It is easy to show through integration by parts of (7) and using $M_2(T) = M_{1,\underline{\text{w}}=T_p}$ that $T(\underline{\text{w}}= T_3) = T_3{}^{22}$.

[21] Earned by workers who got their job while single.

[22] In fact it was done in Section 2 with the difference that Section 2 uses η_{vp}instead of η_p

An unemployed worker's problem in this environment can be presented in a way more convenient for my purposes in this section, as it is more familiar to the concept of a corner solution. The payoff of an unemployed worker is described by V_0 as in (2) but with $\underline{\mathrm{w}}= T_3$. If the worker decides to accept any offer with wage $x \geq R$, then, the worker's problem is $\underset{R}{Max} \quad V_0$ subject to:

$$i) F(w) \text{ as given in Section 1}$$

$$ii) V_1(x), V_2(x) \text{ and } V_3(x) \text{ as given in Section 2}$$

$$iii) \quad R \quad \leq \quad T_p, \bar{w} = T_3(1-g) + gp$$

This problem is analogue to the one solved in Section 1, and therefore yields $R^* = R_w$ as in equation (3)

$$R^*(X) = b + \frac{k_0 k_m m \left[1 - \frac{(R_w - T)(-p + R_w)}{2(p-T)^2}\right]}{1 + k_m} - k_0 \Lambda(-p + R_w)$$

Because in the P equilibrium $\underline{\mathrm{w}}= T = T_p$, I impose this to get

$$R^*(X, \underline{\mathrm{w}} = T = T_p) = b + \frac{k_0 k_m m}{1 + k_m} + \frac{k_0 ln(2)(p - X)}{2(1 + \rho_p(1 - ln(2)))} = R^e(X)$$

Proposition 4. Assume $X_{b'} < X < X_b$, where

$$X_{b'} = \frac{2b\left[g + \Phi\rho_p\right]}{(1 + k_0\Lambda)g} + \frac{(k_0\Lambda g - \rho_p\Phi)p}{(1 + k_0\Lambda)g}$$

and X_b has been defined above. Then an equilibrium exists where $R = T = T_p$.

Proof. See appendix. For further reference, notice that $X_{b'} = X_b(m = 0, \rho_{vp} = \rho_p)$

In the Picky Equilibrium, X is high but not too high and so is MPs reservation wage, T. Workers always find it optimal to increase their reservation wage just the required amount in order to be marriageable when employed, i.e. until $R = T$. Opposite to the VP equilibrium:

i) When the P equilibrium obtains, there is no correlation between wages and marital status, there is only a correlation between employment status and marital status

ii) An increase in X (that does not take X above X_b) will make MPs more picky and they will increase their reservation wage T. As a result, workers will increase the reservation wage in the same amount in order to become marriageable when employed. This is clear by just looking at the equilibrium $T = R = T_p$

iii) As the value workers give to marriage increases (an increase in m that keeps $X_b < X < X_a$) this has no effect on worker's reservation wage. This is clear by just looking at the equilibrium $T = R = T_p$. This occurs because workers are marriageable in equilibrium anyway.

4 The Smitten Equilibrium (S).

As opposed to the VP equilibrium, in the S equilibrium marriage partners are willing to marry any worker, regardless of its employment status or wage earned.

Workers.

The payoff of an unemployed and single worker is described by

$$rV_0 = b + \lambda_0 \int_{\underline{w}}^{\bar{w}} \left[\max(V_2(x), V_0) - V_0\right] f(x)dx + \lambda_m(V_0' - V_0) - \delta V_0 \qquad (8)$$

where V_0' is the payoff of being married and unemployed. In (8) above, the worker enjoys unemployment benefit b. Upon a contact with a firm (at rate λ_0), the worker accepts the job and is marriageable since all workers are marriageable. The distribution of wages faced by single workers is $F(w)$. At rate λ_m, an MP is contacted and marriage occurs. The worker dies at rate δ. The payoff of being married and unemployed is given by

$$rV_0' = b + m + \lambda_0 \int_{\underline{w}_m}^{\bar{w}_m} \left[\max(V_3(x), V_0) - V_0\right] h(x)dx - \delta V_0 \qquad (9)$$

In 9) above, the worker enjoys unemployment benefit b and the utility derived from marriage. Upon contact with a firm, a job is accepted and the worker becomes married and employed. The distribution of wages faced by married workers is $H(w)$. Notice, the minimum wage for married workers is given by $\underline{w}_m$, not $\underline{w}$. Arguments analogous to those applied to (1) imply (8) and (9) can be written as[23]

$$rV_0 = b + \lambda_0 \int_R^{\bar{w}} \left[V_2(x) - V_0\right] f(x)dx + \lambda_m(V_0' - V_0) - \delta V_0 \tag{10}$$

$$rV_0' = b + m + \lambda_0 \int_{R_m}^{\bar{w}_m} \left[V_3(x) - V_0\right] h(x)dx - \delta V_0 \tag{11}$$

[24].Integration by parts of (10) and (11), using $V_2(x)$ and $V_3(x)$, $F(w)$ with $R = \underline{w}$, $\bar{w} = gp + (1-g)\underline{w}$ and $H(w)$ with $R_m = \underline{w}_m, \bar{w}_m = gp + (1-g)\underline{w}_m$ yields

$$R = R_m = \frac{b + k_0 \Lambda p}{1 + k_0 \Delta} \tag{12}$$

Notice these two reservation wages are independent of m. This is because workers need not worry about marriageability when determining their reservation wage, since they are always marriageable. This is also the intuition for the equality $R = R_m$.

Marriage Partners.

The payoff of a single MPs must be adjusted because now single MPs marry any worker met, regardless of employment status. Hence

$$rM_1 = X + \eta_s \int_R^{\bar{w}} \left[M_2(x) - M_1\right] g(x)dx + \eta_s^u(M_0 - M_1) - \delta M_1$$

[23]Given a wage offer w has been received by a single worker, $\frac{\partial V_1(w)}{\partial w} > 0$ and $\frac{\partial V_0}{\partial w} = 0$. Then, the standard definition of a reservation wage implies $V_1(R) = V_0$, $w \gtrless R \Rightarrow V_1(w) \gtrless V_0$. Given a wage offer w has been received by a married worker, $\frac{\partial V_3(w)}{\partial w} > 0$ and $\frac{\partial V_0'}{\partial w} = 0$. Then, the standard definition of a reservation wage implies $V_1(R_m) = V_0'$, $w \gtrless R_m \Rightarrow V_3(w) \gtrless V_0$.

[24]Recall the assumption that divorce is infinitely costly. Hence, whatever wage the married worker accepts, he will remain married.

At rate η_s an employed marriageable worker is contacted, and the distribution of wages earned is $G(w)$[25]. Marriage occurs and new MP's status is "married to employed worker earning w", with payoff $M_2(w)$ (as given by (5)above). At rate η_s^u, an unemployed worker is contacted. Marriage occurs yielding new status as "married to an unemployed worker", with payoff M_0 given by:

$$rM_0 = b + \lambda_0 \int_{R_m}^{\bar{w}_m} [M_2(x) - M(u)]\, h(x)dx - \delta M_0$$

In this equation, the MP's unemployed partner contacts a firm at rate λ_0, which offers a wage w such that $R_m \leq w \leq \bar{w}$ and distributed according to $H(w)$[26].

Integration by parts of M_0 and M_1 above, using $G(x), H(x)$,as defined before, $M_2(w)$ as in (5), and $R = \underline{\mathrm{w}}$, $\bar{w} = (1-g)R + gp$ and $R_m = \underline{\mathrm{w}}_m$, $\bar{w}_m = (1-g)R_m + gp$ yields

$$(r + \delta + \lambda_0)M_0 = b - \lambda_0 \frac{(R(1-g) + gp)}{2(r + \partial)} - \frac{(1-g)h(p-R)}{g(r+\partial)} \tag{13}$$

$$(r + \eta_s + \eta_s^u + \partial)M_1 = X + \eta_s^u M_0 + \frac{p\eta_s \Phi - \eta_s(1-g)ln(1-g)R}{g(r+\delta)} \tag{14}$$

Proposition 5.

The S Equilibrium obtains if $M_0 > M_1 \iff X < X_c$ where$X_c(\rho_s = \rho_p) = \frac{2b\left[g + \Phi\rho_p\right]}{(1+k_0\Lambda)g} + \frac{(k_0\Lambda g - \rho_p\Phi)p}{(1+k_0\Lambda)g}$

$$X_c = X_{b'}(\rho_p = \rho_s) = \frac{2b\left[g + \Phi\rho_s\right]}{(1+k_0\Lambda)g} + \frac{(k_0\Lambda g - \rho_s\Phi)p}{(1+k_0\Lambda)g}$$

Proof. *See Appendix.* For further reference, notice that $X_c = X_b(m = 0, \rho_{vp} = \rho_s)$.

[25] The distribution of wages *earned* by single-employed workers.

[26] Because he is married. Hence, he faces wage distribution $H(w)$ (not $G(w)$) with minimum wage R_m (not R).

5 Matching and Steady State.

To keep things simple, I use quadratic matching in the marriage market and cloning of single MPs. I normalise the number of single MPs to λ_m, and assume that a new marriage partner comes into the market every time one gets married or dies, so as to maintain that stock constant.

Workers can be in either of fives states: u_s is the total number of workers who are single and unemployed , e_s are single and employed earning a marriageable wage $w \geq T$, u_m are married and unemployed; e_m are married and employed and e_{nm} are employed and not marriageable. I assume that a worker comes into the market as single and unemployed every time a worker dies, whatever its state, and normalise so that $u_{s,i}+e_{s,i}+u_{m,i}+e_{m,i}+e_{nm,i} = 1$, where $i = vp, p, s$

Smitten Equilibrium.

Unemployed single workers. The flow in is given by those who replace dead workers (δ). The flow out is given by those workers in this stock who die, marry or find a job. Hence, steady state requires $\delta = u_{s,s}(\delta + \lambda_m + \lambda_w) \Rightarrow u_{s,s} = \frac{\delta}{\delta+\lambda_m+\lambda_w}$.

Employed single workers. The flow in is given by those workers who are unemployed and single and find a job. The flow out is given by those in this stock who die and those who marry after contacting a MP. Hence, the stock $e_{s,s}$ is constant if $u_{s,s}\lambda_w = e_{s,s}(\lambda_m + \delta)$, which implies $e_{s,s} = \frac{\delta}{\delta+\lambda_m+\lambda_w}\frac{\lambda_w}{\lambda_m+\delta}$.

Unemployed married workers. The flow in is given by those workers who are single and unemployed and marry after contacting a MP. The flow out is given by those in this stock who die or find a job. Hence, steady state requires $u_{s,s} = u_{m,s}(\delta + \lambda_w) \Rightarrow u_{m,s} = u_{s,s}\frac{\lambda_m}{\delta+\lambda_w} = \frac{\delta}{\delta+\lambda_m+\lambda_w}\frac{\lambda_m}{\delta+\lambda_w}$

Employed married workers. The flow in is given by those workers who are employed and single and marry after contacting a MP; and by those married and unemployed who find a job. The flow out is given by those in this stock who die. Hence, steady state requires $e_{s,s}\lambda_m + u_{m,s}\lambda_w = e_{m,s}\delta \Rightarrow e_{m,s} = e_{s,s}\frac{\lambda_m}{\delta} + \frac{u_{m,s}\lambda_w}{\delta} \Rightarrow e_{m,s} = \frac{\delta}{\delta+\lambda_m+\lambda_w}\frac{\lambda_w}{\lambda_m+\delta}\frac{\lambda_m}{\delta} + \frac{\delta}{\delta+\lambda_m+\lambda_w}\frac{\lambda_m}{\delta+\lambda_w}\frac{\lambda_w}{\delta}$

Employed non marriageable workers. All workers are marriageable in the S equilibrium, so $e_{nm,s} = 0$

Given that I use a quadratic meeting function, this means that $\eta_s^u = \frac{\delta}{\delta+\lambda_m+\lambda_w}\frac{\lambda_m}{\delta+\lambda_w} = u_{s,s}$ and $\eta_s = \frac{\delta}{\delta+\lambda_m+\lambda_w}\frac{\lambda_w}{\lambda_m+\delta} = e_{s,s}$.

Very Picky Equilibrium.

The difference compared to the S equilibrium is that unemployed workers cannot get married, and not all employed workers can get married, but only those that earn $R \geq T$.

Unemployed single workers. The flow in is given by those who replace dead workers. The flow out is given by those workers in this stock who die or find a job. Hence, steady state requires $\delta = u_{s,vp}(\delta + \lambda_w) \Rightarrow u_{s,vp} = \frac{\delta}{\delta+\lambda_w}$.

Employed single workers. The flow in is given by those workers who are unemployed and single and find a job with a marriageable wage. The flow out is given by those workers in this stock who die or marry after contacting a MP. Hence, steady state requires $u_{s,vp}\lambda_w(1 - F(T)) = e_{s,vp}(\lambda_m + \delta)$ which substituting out $u_{s,vp}$ implies $e_{s,vp} = \frac{\delta}{\delta+\lambda_w}\frac{\lambda_w(1-F(T))}{\lambda_m+\delta}$.

Unemployed married workers. Unemployed workers are not marriageable in this equilibrium so $u_{m,vp} = 0$

Employed married workers. The flow in is given by those workers who are single and employed marry after contacting a MP. The flow out is given by those in this stock who die. Hence, steady state requires $e_{s,vp}\lambda_m = e_{m,vp}\delta \Rightarrow e_{m,vp} = e_{s,vp}\frac{\lambda_m}{\delta} \Rightarrow e_{m,vp} = \frac{\delta}{\delta+\lambda_w}\frac{\lambda_w(1-F(T))}{\lambda_m+\delta}\frac{\lambda_m}{\delta}$.

Employed non marriageable workers. The flow in is given by those workers who are unemployed and single and accept an job with a unmarriageable wage. The flow out is given by those workers in this stock who die. Hence, steady state requires $u_{s,vp}\lambda_w F(T) = e_{nm,vp}\delta$, which implies $e_{nm,vp} = \frac{\lambda_w F(T)}{\delta+\lambda_w}$.

Again, given the quadratic meeting technology in the marriage market, this means that $\eta_{vp} = \frac{\delta}{\delta+\lambda_w}\frac{\lambda_w(1-F(T))}{\lambda_m+\delta} = e_{s,vp}$.

Lemma 4. $X = X_b$ implies $X_b = X_b(\eta_{vp} = \eta_{vp}^*)$ where $\eta_{vp}^* = \frac{\delta}{\delta+\lambda_w}\frac{\lambda_w}{\lambda_m+\delta} > \eta_s$

Proof. *See appendix.*

Picky Equilibrium.

Because in the P equilibrium $R = T$ and unemployed workers are not marriageable, all stocks are as in the VP equilibrium but with $F(T) = 0$.

The proposition below summarises the information of Propositions 3,4 and 5. I use it to introduce the next section that deals with a mixed strategy equilibrium.

Summary Proposition 6. For $0 < m < m_a$:

If $X_b(\eta_{vp} = \eta^*_{vp}) < X < X_a$, an equilibrium obtains where $R < T$, and $R_1 < R \leq R_2, T_3 < T \leq T_4$

If $X_{b'} < X \leq X_b(\eta_{vp} = \eta^*_{vp})$, an equilibrium obtains where $R = T = T_3$.

If $X < X_c$, an equilibrium exists where $R = T = \frac{2b+k_0 ln(2)p}{2+k_0 ln(2)}$.

6 A mixed strategy equilibrium.

Lemma 5. Assume $0 < m < m_a$. Then $X_c < X_{b'}$.[27]. There is a gap that separates the values of X so that he Smitten and the Picky equilibrium obtain respectively.

Proof. *See appendix*

The intuition for the gap stated in Proposition 5 is as followsLemma 5: Assume MPs marry all employed workers, regardless of their wage, and single unemployed workers decide on a reservation wage R based on this. Now assume that MPs start marrying only workers earning a wage $T > R$. If unemployed workers will still accept jobs at wages $w < T$, they must be compensated for the loss of marriageability. This implies that their reservation wage increases from R. Notice that the shape of the wage distribution is not affected. Hence, a higher R implies a smaller relative wage inequality. Rather than reinforcing the MPs' behaviour of marrying only high earners, this would give them further incentive to marry all employed workers, as

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with a smaller relative wage inequality the option of continued search is less attractive.

Following Lemma 5, this section shows that a mixed strategy equilibrium obtains if $X_c \leq X \leq X_{b'}$. In the mixed strategy equilibrium, MPs marry *all* employed workers and marry unemployed workers with probability γ. Hence, the value of a MP that is single is given by

$$rM_1 = X + \eta_m \int_R^{\bar{w}} \left[M_2(x) - M_1\right] g(x)dx + \eta_m^u \gamma (M_0 - M_1) - \delta M_1 \qquad (15)$$

where M_0 is value of marriage to an unemployed worker. Subscript m identifies the mixed strategy equilibrium. and η_m^u is the number of unemployed workers. M_0 is described by the following equation

$$rM_0 = b + \lambda_0 \int_{R_m}^{\bar{w}_m} \left[M_2(x) - M_0\right] h(x)dx - \delta M_0 \qquad (16)$$

where $R_m = \frac{b+k_0\Lambda p}{1+k_0\Delta}$. Recall the distribution of wages offered to married employers is given by $I(x)$, which implies that the distribution of wages faced by married workers in their job search is $H(x)$. The mixing strategy is rational only if $M_1 = M_0$. Integration by parts and algebraic manipulation of (15) and (16) above shows that this occurs only if $X = X_{c'}$, where

$$X_{c'} = \frac{2b\left[g + \Phi\rho_m\right]}{(1 + k_0\Lambda)g} + \frac{(k_0\Lambda g - \rho_m\Phi)p}{(1 + k_0\Lambda)g}$$

The steady state equations in the mixed strategy equilibrium are as follows: The stock $u_{s,m}$ remains constant if $\delta = u_{s,m}(\delta + \lambda_m\gamma + \lambda_w) \Rightarrow u_{s,m} = \frac{\delta}{\delta+\lambda_m\gamma+\lambda_w}$. The steady state equation for $e_{s,m}$ is given given by $e_{s,m} = \frac{u_{s,m}\lambda_w}{\lambda_m+\delta}$, which substituting out $u_{s,m}$ implies $e_{s,m} = \frac{\delta\lambda_w}{(\delta+\lambda_m\gamma+\lambda_w)(\lambda_m+\delta)}$ Stock $u_{m,m}$ remains constant if $u_{s,m}\lambda_m\gamma = u_{m,m}(\delta + \lambda_w)$, which means $u_{m,m} = \frac{\delta}{\delta+\lambda_m\gamma+\lambda_w}\frac{\lambda_m\gamma}{(\delta+\lambda_w)}$. Stock $e_{m,m}$ remains constant $e_{s,m}\lambda_m + u_{m,m}\lambda_w = e_{m,m}\delta \Rightarrow e_{m,m} = e_{s,m}\frac{\lambda_m}{\delta} + \frac{u_{m,m}\lambda_w}{\delta} \Rightarrow e_{m,m} = \frac{\lambda_w\lambda_m}{(\delta+\lambda_m+\lambda_w)(\lambda_m+\delta)} + \frac{\lambda_m\gamma\lambda_w}{(\delta+\lambda_m\gamma+\lambda_w)(\delta+\lambda_w)}$. In this equilibrium, $e_{nm,m} = 0$. This means that $\eta_m = \frac{\delta\lambda_w}{(\delta+\lambda_m\gamma+\lambda_w)(\lambda_m+\delta)} = e_{s,m}$.

Proposition 7. A mixed strategy equilibrium obtains if $X_c \leq X \leq X_{b'}$

Proof. *See appendix.*

7 Conclusion.

I obtain the equilibria in a model in which a search labour market and a matching marriage market interact. The economy is populated by ex-ante homogeneous workers, ex-ante homogenous firms, and ex-ante homogeneous marriage partners. Workers simultaneously search for firms in order to work and for marriage partners in order to marry. Firms post wages to attract workers; and marriage partners look for workers in order to marry. When married, I assume that workers receive a pre-determined flow utility, and that marriage partners derive utility equal to the worker's wage. I show that the so called "married wage premium" or, more generally, a correlation between men's wages and marital status, can emerge as an equilibrium result of having search frictions both in the labour and the marriage market[28]. I do not know of another model that analyses the equilibrium interaction of a search market (the labour market) and a matching market (the marriage market), which I see as the main theoretical contribution of the paper.

In order to obtain clean analytical results, I use some assumptions the removal of which seems interesting and is the basis of current research. For example, if divorce is allowed, the model seems to yields empirically valid predictions not only about the married wage premium, but also about the "divorced wage premium". Namely, that divorce men enjoy a wage premium smaller than married men, but still positive over never married men. When an unmarried and unemployed worker accepts an unmarriageable wage he looses the option to get married in the future (or what I have termed "marriageability"). When a married and unemployed worker accepts an unmarriageable wage he is divorced by his partner, thereby loosing marriage itself,

[28]Not having to resort to the traditional explanations given for the existence of the married wage premium.

which is more valuable than the option of a future marriage. Hence, provided both have a reservation wage lower than that of marriage partners[29], the reservation wage of married workers is higher than the reservation wage of unmarried workers, as they loose more when accepting an unmarriageable wage.

I assume that single marriage partners enjoy a predetermined flow utility, which I call X. Amongst other things, X could be interpreted as the option of marrying differently skilled workers. Preliminary research using this interpretation yields interesting insights on which type of workers should enjoy higher married wage premia. In particular, in a situation where there are differently skilled workers and high shilled workers are more likely to earn high wages, a marriage partner could accept marriage to unemployed high skill workers (expecting a high wage when the worker finds a job); but not to low skill workers employed at a wage in the low end of the distribution. Hence, a correlation exists between wages and marital status for low skill workers, but not for high skill workers.

8 Appendix.

Proof of Proposition 1. Taken together, statements $a) - e)$ below imply $R_w(T)$ is always higher than b for $R_w < T < T_2$, it is downward sloping, continuous and concave when $R_w = T = T_2$.

$a)$ From (3), it follows that $R_w = b$ if $T = T_a$, where

$$\begin{aligned} T_a &= \frac{-2k_m Mgb + (2\Lambda gp(1+k_m) + k_m m(1+g^2))(-p+b) + (-p+b)\sqrt{\Gamma}}{2g(\Lambda(1+k_m)(-p+b) - k_m m)} \\ \Gamma &= (-Mk_m(-1+g)^2(4\Lambda g(1+k_m)(-p+b) - (g+1)^2 k_m m) > 0 \end{aligned}$$

Further, $T_a \geq T_2$ iff $m < m_a$ as in the body of the paper.

Implicitly differentiating (3) it is easy to show that

[29] which can be shown to happen in some equilibria.

$b)$ $\frac{\delta R_w}{\delta T}$ when $R_w = T = R_1$ is $\frac{\delta R_w}{\delta T}_{(R_w=T=R_1)} = \frac{(1-g)^2 k_0 k_m m}{k_0 k_m m(1-g+g^2)+g(1+k_m)(-p+b)}$. Further $\frac{\delta R_w}{\delta T}_{(R_w=T=R_1)} < 0$ when $m < m_a$.

$c)$ $\frac{\delta R_w}{\delta T} = 0$ if $T = 2R_w - p > \bar{w}$. Therefore $\frac{\delta R_w}{\delta T} \neq 0$ when $R_w < T < \bar{w}$

$d)$ $\frac{\delta R_w}{\delta T}$ exists iff $m \neq m_b = \frac{g(T-p)^2(1+k_m)(k_0\Lambda+1)}{k_m k_0(1-g)^2(T+p-2R_w)^2}$. It is easy to show that $m_a < m_b$ when $R_w < T < T_2$, .so R_w is a smooth function in that range.

$e)$ $\frac{\delta [R_w]^2}{\delta^2 T} < 0$ iff $m < m_c = \frac{2g(p-T)^2(1+k_m)(1+k_0\Lambda)(T-3R+2p)}{k_0 k_m(T+p-2R)^3(g-1)^2}$, and it is easy to show that $m_a < m_c$ when $R_w < T < T_2$.

Proof of Proposition 2. Items $i)$ and $ii)$ in Proposition 2 follow directly from (6). Item $iii)$ is the consequence of $a) - c)$ below:

$a)$ From (6), $\frac{\partial R_{mp}}{\partial T} > 0$ when $R_{mp} = T = R_3$. This is easy to show because

$$\frac{\partial R_{mp}}{\partial T} = \frac{g(T-p) + \rho_{vp} g(R_{mp} - p) - \rho_{vp}(R_{mp} - T)}{\rho_{vp} ln \left[\frac{(p-R_{mp})(1-g)}{p-T}\right](p-T)(1-g)}$$

which evaluated at $R_{mp} = T = R_3$ is $\frac{\partial R_{mp}}{\partial T}(R_{mp} = T = R_3) = \frac{g(1+\rho_{vp})}{\ln(1-g)\rho_{vp}(g-1)}$, and this is positive because $0 < g < 1$.

$b)$ $\frac{\partial R_{mp}}{\partial T} > 0$ when $R_{mp} < T < \bar{w}$. This is easy to show because $a)$ above; because $\frac{\partial R_{mp}}{\partial T} = 0$ only if $T = \frac{gp(\rho_{vp}+1)+\rho_{vp}R_{mp}(1-g)}{g+\rho_{vp}} > \bar{w}$; and $\frac{\partial R_{mp}}{\partial T}$ exists when $R_{mp} < T < \bar{w}$.

$c)$Inverting $\frac{\partial R_{mp}}{\partial T}$ as above to obtain $\frac{\partial T}{\partial R_{mp}}$, one can compute

$$\frac{[\partial T]^2}{\partial^2 R_{mp}} = \frac{\rho_{vp}(1-g)(p-T)\left[-1 + \frac{(p-R_{mp})\rho_{vp}(1-g)\ln\left(\frac{(p-R_{mp})(1-g)}{p-T}\right)}{A_1}\right]}{(p-R_{mp})(g(T-p)+\rho_{vp}(T-\bar{w})}$$

where $A_1 = (g(T-p) + \rho_{vp}(T-\bar{w}) < 0$. $A_1 < 0$ implies $\frac{[\partial T]^2}{\partial^2 R_{mp}} > 0$ iff

$$\left[-1 + \frac{(p-R_{mp})\rho_{vp}(1-g)\ln\left(\frac{(p-R_{mp})(1-g)}{p-T}\right)}{A_1}\right] < 0$$

or $(p - R_{mp})\rho_{vp}(1-g)\ln\left(\frac{(p-R_{mp})(1-g)}{p-T}\right) > A_1$. Because (3) can also be written as

$(p - R_{mp})\rho_{vp}(1-g)\ln\left(\frac{(p-R_{mp})(1-g)}{p-T}\right) = A_1 + g(X-p) > A_1$, this ensures $\frac{[\partial T]^2}{\partial^2 R_{mp}} > 0$ so $T(R_{mp})$ is upward sloping and convex, which means that $R_{mp}(T)$ is upward sloping and concave.

Proof of Lemma 1. Follows immediately from solving the equations $R_4 = R_2$ and $T_4 = T_2$.

Proof of Lemma 2. From inspection (3) it follows that $\frac{\partial R_w}{\partial X} = 0$. From implicit differentiation of (6) it follows that (a) $\frac{\partial R_{mp}}{\partial X} = \frac{g}{\rho_{vp} ln(\left[\frac{(p-R_{mp})(1-g))}{(p-T)}\right](1-g)} > 0$ if $R_{mp} < T < \bar{w}$. (b) $\frac{\partial T}{\partial X} = \frac{(p-T)g}{g(p-T)+\rho_{vp}(w\max - T)} > 0$ if $R_{mp} < T < \bar{w}$. (c) $\frac{\partial T_4}{\partial X} = 1 > 0$ and $\frac{\partial T_4}{\partial X} = \frac{1}{1-g} > 0$.

Statements $(a) - (c)$ above imply that as X declines, the graph of $R_{mp}(T)$ in the R_{mp}, T space shifts to the left. Starting at $X = X_a$, a small enough decline in X yields a situation as depicted in Figure 4.

Proof of Lemma 3. *Follows directly by using T_1 and T_3.*

Proof of Proposition 3. *Follows immediately from Lemmas 1-3.*

Proof of Proposition 4. It is straightforward to show that the optimal reservation wage chosen by an MP is $T(\underline{\text{w}}= T_p) = T_p$. I must also show that MPs do not have an incentive to marry unemployed workers. Because the relevant distribution of wages faced married unemployed workers is $H(x)$ the value of marriage to an unemployed worker is given by $rM_0 = b + \lambda_0 \int\limits_{R_m}^{\bar{w}_m} [M_2(x) - M_0]\, h(x)dx - \delta M_0$, where $R_m = \frac{b+k_0\Lambda p}{1+k_0\Delta}$ as shown in the solution to Equation (11) for the Smitten Equilibrium. Simple manipulation of $M_{1,\underline{\text{w}}=T_p}$ and of M_0 shows that $M_{1,\underline{\text{w}}=T_p} \geq M_0$ if and only if $X \geq X_{b\prime}$ as in Proposition 5. Now consider the problem of an unemployed worker as described in this subsection. I first obtain R^* and evaluate it when $\underline{\text{w}}= T = T_p$ to obtain $R^*(X, \underline{\text{w}} = T = T_p) = R^e(X)$. It is easy to show that $R^e(X)$ is downward sloping and continuous in the range $X_{b\prime} \leq X < p$. Also, one can show that $R^e(X_b) = T_p$. Hence, for $X \geq X_b$ we have $R = R^e(X) \leq T_p$, and the equilibrium breaks. For $X < X_b$, then $R^e(X) > T_p$, so workers reach a corner solution where $R = T_p$.

Proof of Proposition 5. Follows immediately from equations (13) and (14). For further reference, notice that $X_c = X_b(m = 0, \rho_s = \rho_{vp})$.

Proof of Lemma 4. $X = X_b$ implies $T_1 = T_3 = R_3 = R_1$. Hence, in equilibrium, $T = R$ and $F(T) = 0$; and this implies $\eta_{vp} = \eta^*_{vp}$.

Proof of Lemma 5. Take $X_b(\eta_{vp} = \eta^*_{vp})$ and X_c as given in Lemma 4 and Proposition 4 respectively. Assume for a moment that $\eta_s = \eta^*_{vp}$. Then $X_c = X_b(m = 0) < X_b(m > 0)$. Further $\frac{\partial X_c}{\partial \eta_s} > 0$ and $\eta_s < \eta^*_{vp}$. This necessarily implies that $X_c < X_b(\eta_{vp} = \eta^*_{vp})$ for $m \geq 0$.

Proof of Proposition 7. It is easy to show that $i)$ $\gamma = 0$ implies $\eta_m = \eta^*_p$ which implies $X_{c'} = X_{b'}$; $ii)$ $\gamma = 1$ implies $\eta_m = \eta_s$ which implies $X_{c'} = X_c$; and that $iii)$ $\frac{\delta X_{c'}}{\partial \gamma} < 0$.

9 References

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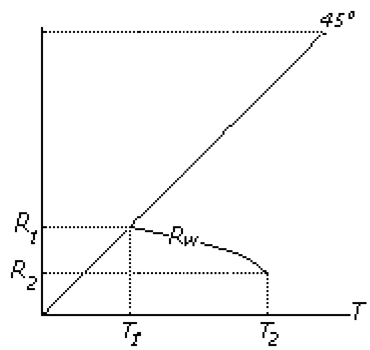

Figure 1: R_w when
$Rw < T < \bar{w}$

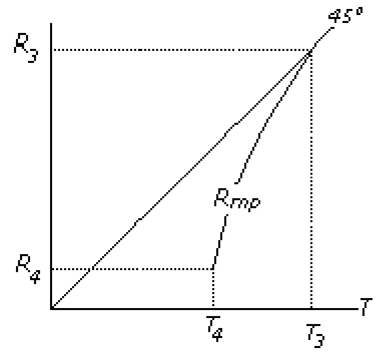

Figure 2: R_{mp} when
$R_{mp} < T < \bar{w}$

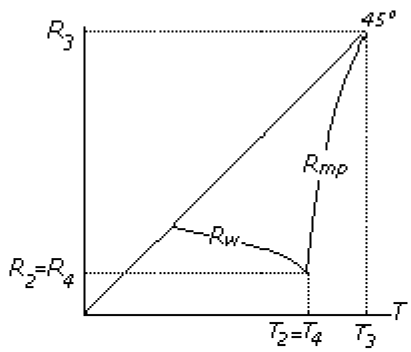

Figure 3: R_w and R_{mp} when
$X = X_a$

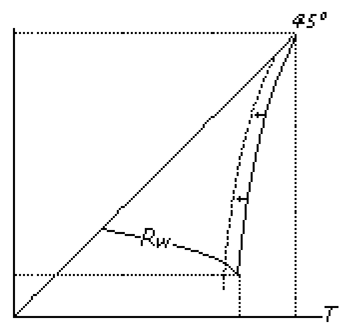

Figure 4: R_w and R_{mp}
when $X = X_a - \epsilon$

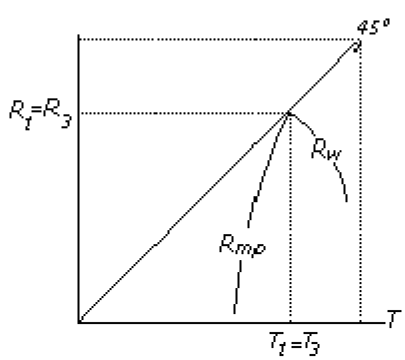

Figure 5: R_w and R_{mp} when
$X = X_b < X_a$