

Marriage, Employment Participation and Home Production in Search Equilibrium

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Abstract

We provide a model where singles choose spouse considering prospects of employment and income, while a married workers decisions on home production and labor participation, as well as their bargaining power for wages, are affected by the employment status and conditions of their spouse. This double interaction between the marriage and labor markets is affected by search frictions in both. We find that couples with different combinations of productivities have different strategies regarding labor participation. When the search for mates is easy, people marry others with very similar productivity, and the behaviour of couples in the labor market is symmetric. When finding mates is hard, people could marry others very different from themselves, and if so their labor market behavior is going to be asymmetric. Married workers get better job offers when their spouses are employed, and in some equilibria a person may search for transitory jobs with the sole purpose of raising the long-term wages of their spouse. Firms may respond by offering some very productive individuals wages that secure that their spouses stay at home. Whether they follow that strategy or not may be a matter of multiple equilibria, depending on parameter values.

1 Introduction

In any labor market there is heterogeneity in workers wages and labor participation rates. Part of it is explained, of course, by differences in productivity: more productive individuals are likelier to pursue work and tend to receive

a better wage. But, in general, other circumstances may also matter. In particular, the employment status, prospects and wages of one's spouse may affect whether one seeks for work, or how much does one earn. Married to a high earner, one is likelier to engage in home production, or to be more selective about which jobs to accept, or to bargain oneself into higher remunerations. Interestingly, since a person can get pretty clear indications about a marital partner's earning potential while pondering on the possibility of marriage, then not only their career is affected by the productive features of their spouse, but their choice of spouse is also affected by their potential careers.

In this paper, we develop a model where agents go first to a marital market and then a labor market. Agents choose their spouses taking into account their expected earnings, and once married the couple makes joint decisions regarding job search and wage bargaining. Hence, the two-directional interaction described above is brought forward. Furthermore, spouses can collaborate not only by working and sharing their income, but also by specializing, one in market work, and the other in home production.

In a simple version of the model, where agents work independently rather than as employees, we find that in equilibrium there are several kinds of couples, that differ in their optimal labor market strategies, which vary with the combination of productivities of the two spouses. In equilibrium, there is a positive correlation in earning potentials among spouses, and when frictions in the marriage market are small, this correlation is very tight. Then, the majority of the population will belong to two classes, that are symmetric in the sense that both spouses have the same labor-participation strategy: a high class of very productive individuals married to other very productive individuals, both staying always in the job market and generating two incomes, while sacrificing home production; also, a middle class of agents with intermediate productivity, who also marry only their similars, and allocate the work burden in the couple symmetrically: when unemployed both seek, and once either one finds a job the other one stays at home. When the frictions in the marriage market get larger, there will also be other types of couples made of spouses that are very different in their productivities, and whose best strategy is asymmetric. Some of those constitute a low class where the lower productivity spouse stays always at home, and the other is always in the job market. There is also a fourth class, where the optimal strategy is that the lower productivity spouse stays at home if their partner is employed (like in the low class), and looks for work if their partner is unemployed (like

in the middle class), while the high productivity partner never drops off the market, so they occasionally have two incomes (like in the high class).

We then expand the model, with firms that bargaining with potential employees over wages. Again, we find several results. First, because the bargaining position of a worker is stronger when their spouse are making a higher income, it turns out that married people's income is correlated in two ways: not only the more productive you are the more productive a spouse you seek, but also, given productivities, the better paid you are, the higher pay your spouse can bargain. Second, we find equilibria where the spouse with lower earning potential transitorily goes to the job market, just to improve with their salary the bargaining power of their partner. Third, for some parameter values, it may be an equilibrium for the more productive spouse to stay at home and the less productive one to be the sole bread winner, obviously an inefficient arrangement. Fourth, we find equilibria where employers prefer to reduce turnover by "bribing" their employee's spouse to stay at home. When this happens, the wage distribution is compressed relative to the productivity distribution. Fifth, for some couples there may be multiple equilibrium wages and job search strategies.

Other authors have looked previously at the interaction between the marriage and labor markets. In terms of theoretical work, we expand on Violante (2012), who shows how reservation wages are affected by marital status and joint search. Bonilla and Kiraly (2013) generate a wage premium for married workers purely as a result of combining the search frictions in both markets. Along similar lines, Jaquemet and Robin (2013) study individual labour supply with a frictional marriage market. Regarding empirical work, Schwartz and Mare (2005) convincingly documents that as the search technology has improved, the positive correlation in earning potential among spouses has increased, raising overall income inequality. This increased symmetry in the human capital that the spouses bring into the household reflects in an increasing similarity in their inputs and home production hours, as has been shown as far back as Cancian et.al (1993). Schwartz (2010) analyzes the data and reaches conclusions about the assortative nature of spouse choices, and about the implicit participation decisions, that interestingly fit our main theoretical results. Korenman and Newmark (1991) is among the first pieces to document empirically a premium in wages for married men; others have elaborated that this relationship is stronger or more robust among married men with working wives. Among them, Jacobsen and Rayak (1996) show the data is consistent with the explanation that the wife's income allows the

husband to search for better jobs. Song (2006) finds that the relationship is positive except when the wife is in management (which perhaps can be interpreted as her being in the labor market permanently rather than temporarily). All these empirical results are consistent with the findings in this paper.

We describe the environment in Section 2, and derive and analyze in Section 3 the equilibrium in a simple case, where there is no firm behavior because workers get all the surplus. In Section 4 we study the equilibrium in the job market in a case where firms have bargaining power and interact strategically with workers, and in Section 5 we derive for that case the behavior in the marriage market, that is, who is willing to marry whom. We conclude in Section 6.

2 The environment

Time is continuous and continues forever. The population is a continuum of measure one of infinitely-lived agents, who discount the future at rate r , and are identical in everything except their productivity p . We denote $F(p)$ the distribution function of p , with $[p, \bar{p}]$ being the range of that distribution.

When young, agents are in a marriage market, where they can search (at a minimal but positive search cost ε) and encounter –through a Poisson process with arrival rate $\hat{\mu}$ – potential partners with whom they could marry. A fraction γ of couples will be compatible, in the sense that –productivity and income aside– they have whatever it is that it takes to form a permanent match. Let $\mu = \hat{\mu}\gamma$.

Upon meeting, a compatible couple also observe each others productivity, and they enter a permanent monogamous relationship if doing so is strictly mutually agreeable, and depart the marriage market; otherwise, they keep searching for another spouse.¹ When a couple marries, two clones of the newlyweds take their place. We assume that, outside of financial considerations, a partner may be suitable or unsuitable for marriage, but there are no varying degrees of suitability beyond that. We also assume there is no gender

¹We are not assuming that productivity and earning potential are the only characteristic that one seeks in a mate. What we are assuming is that those other characteristics that determine compatibility are binary (either I love you or I don't, and if I could love two people, I would love them the same), homogeneously distributed among the population, and uncorrelated with productivity.

(all agents seek spouses out of the same population).² Also for simplicity, we assume that single agents do not work. People first seek spouse and decide on marriage, and only later enter the labor market.

Spouses share income and home production; once married, preferences belong to the couple, not the individual spouses. The value of home production increases with income (that, for instance, enables to acquire household goods that complement home work), but requires that at least one member of the couple is not working. We denote α the marginal effect that income has as an enhancer of home production, which is then $h + \alpha w$. Hence, instantaneous flow utility is

$$U = \begin{cases} w + W & \text{both work and make wages } w \text{ and } W \\ w + h + \alpha w & \text{if one works for wage } w \text{ and the other does not work} \\ h & \text{neither is employed} \end{cases}$$

An unemployed worker that has never left the labor market can search for potential employers, again at a search cost ε ; we assume $\varepsilon > 0$ but look at the limit case where $\varepsilon \rightarrow 0$.³ Firms will be found through another Poisson process, with arrival rate λ . Employed workers cannot search. All employers (firms) are identical and able to generate costlessly as many vacants as they want, and only workers matched with a firm are able to produce. Hence, as long as a job is profitable the firm would never terminate it to make room for others, and its interaction with one potential employee is not affected by quantity or qualities of its other current or potential workers. Upon a match, firm and worker enter a bargaining game to decide the wages, with full information regarding the productivity and working history of the worker and the worker's spouse.

The bargaining game that determines wages is as follows: Upon meeting, the firm offers a wage with probability θ , and the worker with probability

²If we allowed for gender, and assumed all parameters distributed equally between males and females, then the equilibria that we find in the paper would translate into a set of symmetric equilibria, but there could be asymmetric equilibria where expectations that men and women are treated differently become self-fulfilling. Additionally, there could be actual differences in parameters between men and women, also generating asymmetries in the results. While such things may happen and are interesting in their own right, we focus on other matters in this paper, for which ignoring gender seems to be the simplest way to go.

³This simply reduces the degree of indeterminacy of equilibria: whenever they expect that a positive surplus could come out of it, agents search, and if indifferent about the outcome then they unambiguously do not.

$1 - \theta$. If the worker rejects it, then after a small wait of duration Δ the firm makes a final offer with probability $\theta/2$, and the worker with probability $1 - \theta/2$. If this final offer is rejected, the pair separates forever. If either offer is accepted, that contract is binding, and can only be changed by breaking the relationship (there is no renegotiation).

We will look at two options for θ . In Section 3, we assume $\theta = 0$, and the description in this paragraph is just a complicated way of saying that the worker has all the bargaining power, and thus appropriates her entire productivity as if she was an independent worker, so the firm is redundant: not making any profits or relevant decisions. In Section 4, we look at a more interesting case by assuming that $\theta = 1$, allowing firms to make profits and to behave strategically towards the worker.⁴ Once matched, a firm and a worker stay together until, with arrival rate $\delta > 0$, the relationship is broken. For now, we assume that these events are the only cause that can terminate a match, but we generalize this assumption in Section 4.

3 Equilibrium without profits: $\theta = 0$

We work out in this section the labor market choices of couples when $\theta = 0$. This assumption implies that each worker will always be paid exactly his productivity, not more or less, in every job. Firms are not really making decisions; jobs are more like production facilities that the worker encounters and uses, all the same for a given worker. This simple case allows us to grasp some understanding about the choices of labor market participation in the couple, as spouses are affected by their joint enjoyment of income and human capital.

Due to the sequential nature of the problem, we can work out the labor market performance of any possible couple (whether in equilibrium such a couple would exist, instead of one of the partners choosing not to marry the other, is a different issue to be addressed later). With no loss of generality, we will label H the spouse with a weakly higher productivity, and L to the other spouse. Their productivities will be denoted p_H and $p_L \leq p_H$. The value functions that correspond to their circumstances are denoted V_{hl} , where

⁴With $\theta = 1$, the bargaining game is the same one used in Bonilla and Burdett (2010). In both papers, this departure from the more standard Nash bargaining option is justified because in this environment the choice set may not be convex, and hence the axiomatic Nash solution may not exist.

$h = 1$ implies that H is employed and $h = 0$ that H is unemployed, and l is an analogous indicator for the status of L . Needless to say, these functions V_{hl} are specific to each couple, as another pair with different productivities would enjoy different payoffs. Then,

$$\begin{aligned}
rV_{11} &= p_H + p_L + \delta(V_{01} + V_{10} - 2V_{11}) \\
rV_{10} &= (1 + \alpha)p_H + h + \delta(V_{00} - V_{10}) + \lambda\phi_2(V_{11} - V_{10}) \\
rV_{01} &= (1 + \alpha)p_L + h + \delta(V_{00} - V_{01}) + \lambda\phi_3(V_{11} - V_{01}) \\
rV_{00} &= h + \lambda\phi_0(V_{10} - V_{00}) + \lambda\phi_1(V_{01} - V_{00})
\end{aligned} \tag{1}$$

The first equation simply implies that the flow value of both spouses being employed, rV_{11} is equal to their joint income, $p_H + p_L$, plus the arrival rate δ of the loss from the destruction of the high-paying job, $(V_{01} - V_{11})$, plus the arrival of the loss from the destruction of the low-paying job, $\delta(V_{10} - V_{11})$. The second equation that a couple where only H works enjoys income p_H , plus the fruits of home production $h + \alpha p_H$, plus, the arrival of the loss of that job, $\delta(V_{00} - V_{10})$, plus, given that L is searching for a job with probability ϕ_2 , the arrival λ of such a job, which would deliver the net gain $V_{11} - V_{10}$. The other two equations can be understood analogously.

Notice that we are using the variables $\phi_i, i \in \{0, 1, 2, 3\}$ to denote the binary decisions about whether or not to search for a job in different states: for instance $\phi_1 = 1$ implies that the lower-productivity spouse L searches when both spouses are unemployed. For couples to behave optimally, it must be the case that at every chance they only search for a job if it improves their condition, so

$$\begin{aligned}
\phi_0 &= \begin{cases} 1 & \text{if } V_{10} > V_{00} \\ 0 & \text{otherwise} \end{cases} \\
\phi_1 &= \begin{cases} 1 & \text{if } V_{01} > V_{00} \\ 0 & \text{otherwise} \end{cases} \\
\phi_2 &= \begin{cases} 1 & \text{if } V_{11} > V_{10} \\ 0 & \text{otherwise} \end{cases} \\
\phi_3 &= \begin{cases} 1 & \text{if } V_{11} > V_{01} \\ 0 & \text{otherwise} \end{cases}
\end{aligned} \tag{2}$$

Definition 1 For all couples (H, L) an optimal strategy is a combination of values $V = (V_{00}, V_{01}, V_{10}, V_{11})$ and job search probabilities $\phi = (\phi_0, \phi_1, \phi_2, \phi_3)$

that satisfies the Bellman equations (1) and incentive compatibility conditions (2).

The set of possible situations narrows quite a bit thanks to the following:

Lemma 2 *In any optimal strategy, a) $\phi_0 = 1$, and b) $\phi_2 = 1 \implies \phi_1 = \phi_3 = 1$.*

Proof. Recall that there are no mixed strategies so the values $\phi_i \in \{0, 1\}$. a) Clearly, $\phi_0 = 0$ cannot be an optimal strategy when $\phi_1 = 0$, because the couple can raise their income with no sacrifice in home production if at least one of the members gets a job. On the other hand, $\phi_0 = 0$ while $\phi_1 = 1$ cannot be the optimal strategy since it is obviously dominated by $\phi_0 = 1, \phi_1 = 0$. Hence, there are no optimal strategies where $\phi_0 = 0$. b) When $\phi_2 = 1$, the lower-productivity spouse accepts even though the higher-productivity spouse is employed. Then, the option of gaining p_L is worth giving up $h + \alpha p_H$. Then, it has to be that the same option of gaining p_H (which is a higher prize) is also worth giving up $h + \alpha p_L$ (which is a lower sacrifice). Hence, $\phi_3 = 1$. Furthermore, it also has to be that the same option of gaining p_L (the same prize) is worth giving up h (a lower sacrifice). Hence $\phi_1 = 1$. ■

This lemma implies that there are three combinations of ϕ_i that are never optimal strategies. Accordingly, we can also collapse two other combinations into one, since $\phi_1 = \phi_2 = 0$ implies that ϕ_3 does not come up along the equilibrium path. Therefore, there are at most four possible types of optimal strategies, where $\phi = (\phi_1, \phi_2, \phi_3)$ assumes the values $(0, 0, \cdot)$, $(1, 0, 0)$, $(1, 0, 1)$ or $(1, 1, 1)$. To verify in which region of parameter space can one of those values constitute an optimal strategy, one substitutes the candidate values for ϕ_i in the system (1), solves for V_{hl} in that system, and then checks in which combinations of parameters the max operators in the last three equations are solved by the candidate values of ϕ_i . That is the procedure to prove the following:

Proposition 3 *For all possible couples $(p_H, p_L) \in [\underline{p}, \bar{p}] \times [\underline{p}, p_H]$, $\exists!$ $\phi = (\phi_1, \phi_2, \phi_3)$ that is optimal. In particular, for some values p^* , $p^o < p^*$ and $g_i(p_H)$, $i \in \{1, \dots, 4\}$, the optimal strategy is*

a) $\phi = (1, 1, 1)$ if $p_H > p^*$ and $p_L > g_1(p_H)$.
b) $\phi = (1, 0, 0)$ if $p^o < p_H \leq p^*$ and $p_L > g_3(p_H)$ or if $p_H \leq p^o$ and $p_L >$

$g_2(p_H)$.

c) $\phi = (1, 0, 1)$ if $p_H > p^*$ and $p_L \in (g_4(p_H), g_1(p_H)]$, or if $p_H \in (p^\circ, p^*]$ and $p_L \in (g_4(p_H), g_3(p_H)]$

d) $\phi = (0, 0, \cdot)$ if, $p_H \leq p^\circ$ and $p_L \leq g_2(p_H)$, or if $p_H > p^\circ$ and $p_L \leq g_4(p_H)$.
The functions g_i are increasing and linear.

Proof. The procedure, for all combinations ϕ that are not ruled out by Lemma 1, is to solve (1) for V_{hl} given ϕ , and then verify the parameter combinations for which (2) hold.

Consider first the strategy $\phi = (1, 1, 1)$, which requires that $V_{01} \geq V_{00}$, $V_{11} \geq V_{10}$ and $V_{11} \geq V_{01}$. Solving for V in (1) we can translate these inequalities into $p_L \geq g_{111}^1(p_H)$, $p_L \geq g_{111}^2(p_H)$ and $p_L \leq g_{111}^3(p_H)$, where the functions g are all linear and increasing. One can show easily that $g_{111}^2(p_H) < p_H$ implies that $g_{111}^3(p_H) > p_H$ (so g_{111}^3 is redundant) and that $p_H > p^* > 0$ for some p^* . Also, $g_{111}^2(p_H) > g_{111}^1(p_H)$ (so g_{111}^1 is redundant as well). Then, calling $g_{111}^2 = g_1$, we know that $\phi = (1, 1, 1)$ is the best search strategy of the household if and only if

$$p_H > p^* \equiv \frac{h(r + \delta + 2\lambda)}{(1 - \alpha)(r + 2\lambda) + 2\delta}$$

$$p_L > g_1(p_H) \equiv \frac{(h + \alpha p_H)(r + \delta + 2\lambda)}{r + 2(\delta + \lambda) + \alpha\delta}$$

With the same logic, the strategy $\phi = (1, 0, 1)$, requires that $V_{01} \geq V_{00}$, $V_{11} < V_{10}$ and $V_{11} \geq V_{01}$, which again can be translated into $p_L \geq g_{101}^1(p_H)$, $p_L < g_{101}^2(p_H)$ and $p_L \leq g_{101}^3(p_H)$, for some g_{101}^i with similar properties. It simplifies matters to find that $g_{111}^2(p_H) = g_{101}^2(p_H)$. Furthermore, when $p_H > p^*$, the condition on g_{101}^3 is redundant, but there is an interval $(p^\circ, p^*]$ where g_{111}^2 is redundant and g_{101}^3 binding. Finally, the condition for g_{101}^1 is relevant for all $p_H \in (p^\circ, \bar{p}]$, since $g_{101}^1(p^\circ) = g_{101}^3(p^\circ)$. The function g_{101}^1 is what we call g_4 in the text, and the function g_{101}^3 what we call g_3 . Then, we know that $\phi = (1, 0, 1)$ is the best strategy for a couple if their productivities

satisfy

$$\begin{aligned}
p_H &> p^* \text{ and } g_1(p_H) \geq p_L \geq g_4(p_H) \equiv \frac{\lambda(h + \alpha p_H)}{(r + 2\delta)(1 + \alpha) + \lambda} \text{ or} \\
p^* &\geq p_H \geq p^o \equiv \frac{h(r + \delta + \lambda)}{r + (2 + \alpha)\delta + (1 - \alpha)\lambda} \text{ and } g_3(p_H) \geq p_L \geq g_4(p_H) \text{ where} \\
g_3(p_H) &\equiv \frac{p_H [r^2 + r [(3 + \alpha)\delta + 2\lambda] + \delta [(2 + \alpha)\delta + (3 + \alpha)\lambda]] - h(r + \delta)(r + \delta + 2\lambda)}{r^2\alpha + 2r\alpha(\delta + \lambda) + \delta [\lambda + \alpha(\delta + 3\lambda)]}
\end{aligned}$$

The method for the region where the best response is $\phi = (1, 0, 0)$ is similar, and this strategy is optimal in the region where

$$\begin{aligned}
p^* &\geq p_H \geq p^o \text{ and } p_L \geq g_{101}^3(p_H) \text{ (defined above) or} \\
p_H &< p^o \text{ and } p_L \geq g_2(p_H) \equiv \frac{\lambda p_H}{r + \lambda + \delta}
\end{aligned}$$

Finally, the region where the best response is $\phi = (0, 0, \cdot)$, by identical logic, simply requires either $p_H \geq p^o$ and $p_L \leq g_3(p_H)$ or $p_H < p^o$ and $p_L \leq g_2(p_H)$.

■

The following graph illustrates the optimal strategies for different regions in (p_H, p_L) , according to Proposition 1.

It is interesting to note that two of the strategies presented in Proposition 1 are symmetric, in the sense that the actions in the labor market that it requires are the same from both spouses: in $\phi = (1, 1, 1)$, both are always in the job market, and in $\phi = (1, 0, 0)$, both search when neither is employed, and neither one searches when the other is employed. Not surprisingly, these symmetric strategies are optimal when the spouses are very similar in productivity. Meanwhile, strategy $\phi = (1, 0, 1)$ requires H to stay in the job market always while L only if both are unemployed, optimal for intermediate levels of p_L and high levels of p_H . Strategy $\phi = (0, 0, \cdot)$ is characterized by L never joining the job market, and is optimal whenever p_L is low enough.

3.1 Marriage

In the marriage market, compatible, unemployed single people encounter each other at rate μ . Denote $W(p)$ the value for an agent with productivity p of searching in the marriage market. Obviously, for this agent there is a

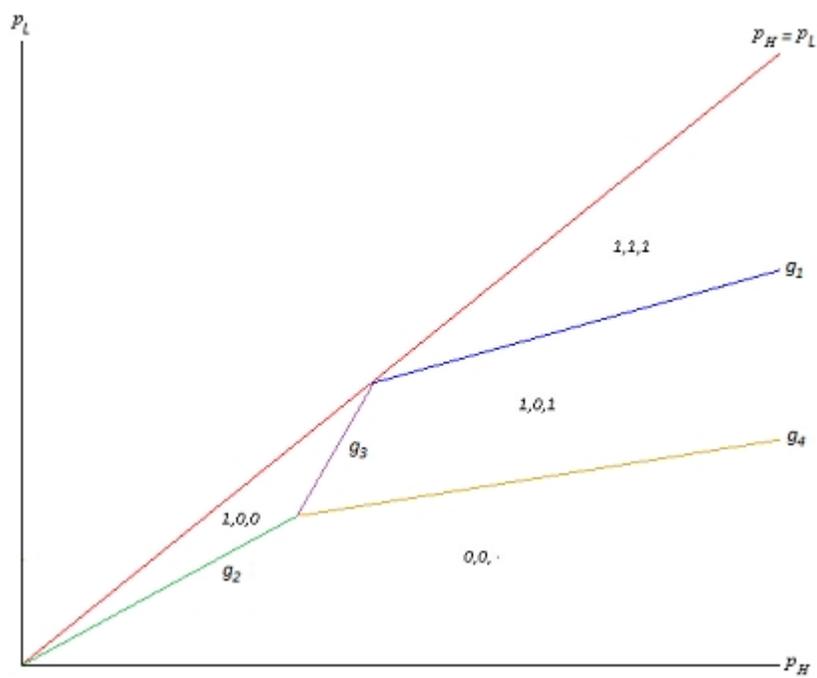


Figure 1: Regions of the space (p_H, p_l) where the different equilibria exist

reservation value, call it $R(p)$, such that she is not willing to marry a spouse, even if compatible, with productivity $\pi < R(p)$. Then

$$W(p) = \mu \int_{R(p)}^{R^{-1}(p)} V_{0,0}(\max\{p, \pi\}, \min\{p, \pi\}) dF(\pi). \quad (3)$$

The bounds in the integral simply imply that a single with productivity p would not accept a marriage proposal from somebody with $\pi \leq R(p)$, nor get one from somebody with $\pi \geq R^{-1}(p)$.

Definition 4 *An equilibrium in the marriage market is a value function $W(p)$ and reservation strategy R such that (3) holds and $W(p) \equiv V_{0,0}(p, R(p))$ for all p .*

It is straightforward to prove the following

Lemma 5 *For any π , the function $V_{0,0}(\max\{p, \pi\}, \min\{p, \pi\})$ is a weakly increasing, piecewise linear, weakly convex function of p , with slope 0 at $p = 0$.*

Proof. For any π , the best job market strategy when $p \sim 0$ is for the agent π to search while the spouse p stays at home. Hence, only π will enter the payoff of the couple, and $\partial W_\pi(p)/\partial p = 0$. Consider what happens as p increases gradually. If \bar{p} is small, then at some value $p = p_1$ the point (π, p_1) is in the border between the region where $\phi = (0, 0, \phi_3)$ is the optimal behavior and the region where $\phi = (1, 0, 0)$ dominates. At that point, both strategies are equivalent, but for slightly larger p the second strategy is dominant, and its value is linear and increasing in p . As p reaches π the role of the spouses is reversed but their behavior remains unchanged, until eventually at some value p_2 the pair (p_2, π) hits the border of the region where strategy $\phi = (1, 0, 1)$ dominates, etc. Depending on the value \bar{p} the sequence of regions visited varies, but in any case as p increases within a region we know that W increases linearly, at any border between regions W has to be continuous, and since the dominated strategy is still feasible it has to be that as a new region is entered the slope of W with respect to p in the *new* region is higher. ■

Of course, because all agents would rank any two (suitable) marriage candidates in the same order, we know from Burdett and Coles (2006) that, in any equilibrium for the marriage market the population will be assorted

in classes, where the members of the top class marry each other and only each other, the members of the second class marry each other and only each other, etc.

Lemma 6 *There is a unique equilibrium of a marriage market, which takes the form of a partition of $[\underline{p}, \bar{p}]$ into n sets S_i , where $S_1 = (R(\bar{p}), \bar{p}]$, $S_2 = (R(R(\bar{p})), R(\bar{p})]$, ... $S_i = (R \circ^i R(\bar{p}), R \circ^{i-1} R(\bar{p})]$, and $S_n = [\underline{p}, R \circ^{n-1} R(\bar{p})]$, where all agents with productivity $p \in S_i$ always marry the first compatible fellow member of S_i that they meet. If μ/r is very low, $n = 1$; n increases with μ/r and the set of married couples $S = \bigcup_{i=1}^n S_i \times S_i$ converges to the 45° line as $\mu/r \rightarrow \infty$.*

Proof. Consider the choices of the most productive agent. If $\mu/r \sim 0$ then $\underline{p} = R(\bar{p})$ and that agent is willing to marry everybody. Consequently, everybody is willing to marry everybody, $n = 1$ and $S_1 = [\underline{p}, \bar{p}]$. For high enough values of μ/r , $R(\bar{p}) > \bar{p}$. In that case, any agent with productivity higher than $R(\bar{p})$ knows that anybody would marry her, and thus that she has the same options as an agent with productivity \bar{p} , and thus the same reservation value: $p > R(\bar{p}) \implies R(p) = R(\bar{p})$. So $S_1 = (R(\bar{p}), \bar{p}]$. Consider now the population whose productivity is less than $R(\bar{p})$. For them, S_1 is unreachable, and thus the maximum they could conceivably marry is $R(\bar{p})$. Agent $R(\bar{p})$ plays the same role in that population as \bar{p} did in the complete population, and thus, if $R \circ R(\bar{p}) > \underline{p} \rightarrow S_2 = (R(\bar{p}), \bar{p}]$. By induction, this process continues until $R \circ^{n-1} R(\bar{p}) = \underline{p}$, defining the final set S_n . ■

Corollary 7 *For arbitrarily large levels of μ/r , all agents have very similar productivity to their spouse, and thus all couples belong to the sets where, in equilibrium, $\phi = (1, 1, 1)$ or $\phi = (1, 0, 0)$. As we consider lower levels of μ/r , and the sets S_i are less numerous but larger, there are bigger productivity differences across spouses, and an increasing fraction of the couples population share the home burdens asymmetrically: $\phi = (1, 0, 1)$ and $\phi = (0, 0, \cdot)$.*

Notice that the dynamics of the marriage market expand the degree of income inequality. High productivity individuals not only are likelier to marry other high productivity individuals, but also the higher their joint productivity, the higher the household's participation rate. Also, given p_H , the lower p_L is the lower the probability that L works, so couples with similar p_H have a family income that is more variable than the productivities themselves.

4 Equilibrium with profits: $\theta = 1$

We look now at the case where $\theta = 1$, which implies that firms have bargaining power, and thus the same worker may encounter different wages offered to him depending on the determinants of the bargaining positions: the spouse's productivity, employment status and wage. We want to focus on situations where there is strong complementarity between home and market production, so we assume $\delta = 0$ and $\alpha = 1$ to ensure that in equilibrium at most one spouse in every couple will work at any given time. Workers can leave a job, but that decision entails a permanent departure from the labor market; notice that with $\delta = 0$ that is the only way of breaking an employment relationship. Also, $\alpha = 1$ assures us that $V_{11} < V_{10}$ and thus that at any given time at most one spouse in each couple will be employed.

Lemma 8 *Given $\alpha = 1$ and $\delta = 0$, a) it is never optimal for both spouses to be employed and keep their jobs; b) if one spouse is employed, the other spouse would only search for a new job if expecting to be offered a higher wage; c) if H is employed at a wage $w \geq p_L$, then L is off the labor market.*

Proof. a) Let the least paid spouse (not necessarily H , because L may have been in a stronger bargaining position than H when their wages got determined) make \underline{w} and the best paid one make $\bar{w} > \underline{w}$. Keeping both jobs guarantees a flow of $\bar{w} + \underline{w}$ for eternity (no exogenous breakups, no search on-the-job), while having the least paid quit (and thus leave the labor market) guarantees a flow of $2\bar{w} + h > \bar{w} + \underline{w}$, also for eternity. b) By virtue of the previous point, any agent accepting a job less paid than their spouse would be immediately quitting and leaving the labor market, and thus losing the option value of search in exchange for a reduction in long-term income, never an optimal move. c) Independent of the outside options, the highest wage that L can be offered is p_L , and by virtue of the previous point that wage would not be acceptable if H is already making $w \geq p_L$. ■

In principle, a newlywed couple may decide that both spouses join the labor market from the outset. On the other hand, they may also choose that initially only one of them searches, delaying the arrival of that first income, but securing the order in which their jobs are going to come. This could happen because, with bargaining, an agent with a better paid spouse has stronger outside options, and gets offered a higher wage. Then, the winning strategy for a couple that desires H to be the long-term breadwinner

of the family could be to have L look a job first, to ensure that H 's wage is higher.⁵ To allow for these possibilities, we will call ϕ_0 the probability that H searches when both partners could and neither is working, and ϕ_1 the analogous probability for L . It is straightforward to show that when L gets a job before H (whether both or only L were searching), it is always optimal for H to search. We will define ϕ_2 as the probability that L searches for a job given that H finds a job first; it is easy to show that if L finds a job first, H will always keep searching.

To complete the notation: if spouse $k \in \{H, L\}$ is the first one to find a job, we denote w_k^1 the wage she gets in equilibrium. If, on the contrary, her partner is already employed at salary w when k finds a job, then her wage will be $w_k^2(w)$. As before, the state where both agents are unemployed and eligible to search will carry a value function V_{00} ; the value of having H employed and earning w while L is not working yet can search is V_{10} , and the reciprocal when it is L earning w and H searching has value V_{01} . We will not need to use V_{11} (since it is never optimal for both spouses to work at the same time, given $\alpha = 1$, $h > 0$).

The Bellman equations write now as

$$\begin{aligned}
 rV_{00} &= h + \lambda\phi_0[V_{10} + -V_{00}] + \lambda\phi_1[V_{01} - V_{00}] & (4) \\
 rV_{01} &= 2w_L^1 + h + \lambda \left[\frac{2w_H^2 + h}{r} - V_{01} \right] \\
 rV_{10} &= 2w_H^1 + h + \lambda\phi_2 \left[\frac{2w_L^2 + h}{r} - V_{10} \right]
 \end{aligned}$$

⁵This is where the assumption that an agent that quits leaves the labor market comes handy. If an agent could quit and stay in the labor market, we could have long cycles where each time H gets a job, L looks for another transitory one with the sole purpose of improving H 's bargaining position in their next job, and raising w_H , until finally after a number of iterations w_H exceeds p_L and the cycle ends. Our assumption still allows for this strategic behavior, by which L only looks for a job to enhance H 's prospects, but limits that cycle to last one iteration.

where the probabilities ϕ_i describe the optimal search strategies, given by

$$\begin{aligned}\phi_0 &= \begin{cases} 1 & V_{10} \geq V_{00} \\ 0 & \text{otherwise} \end{cases} \\ \phi_1 &= \begin{cases} 1 & V_{01} \geq V_{00} \\ 0 & \text{otherwise} \end{cases} \\ \phi_2 &= \begin{cases} 1 & w_H^1 \leq p_L \\ 0 & \text{otherwise} \end{cases}\end{aligned}\tag{5}$$

Recall that when bargaining for a wage, the firm always makes the first offer, knowing that if rejected then, after a wait $\Delta \rightarrow 0$, the definitive counteroffer will be made by either party with probability 1/2. Normally, the firm would offer the wage that makes the worker indifferent between accepting it, or rejecting it and getting a 50-50 chance of making the second and final offer (assuming that if the firm makes the second offer, it would extract all the surplus). For instance, when the firm is bargaining with H over w_H^2 , because L is already employed and the couple is in state V_{01} , the equilibrium offer is defined by the first equation in

$$\begin{aligned}\frac{2w_H^2 + h}{r} - V_{01} &= \frac{1}{2} \left[\frac{2p_H + h}{r} - V_{01} \right] \\ V_{01} - V_{00} &= \frac{1}{2} [\Omega_L(p_L) - V_{00}] \\ \frac{2w_L^2 + h}{r} - V_{10} &= \frac{1}{2} \left[\frac{2p_L + h}{r} - V_{10} \right]\end{aligned}\tag{6}$$

The left hand side represents the gain from the worker of accepting that offer: it will earn w_H^2 forever, while her spouse L stays at home, securing a permanent flow utility of $2w_H^2 + h$, which as a discounted value $\frac{2w_H^2 + h}{r}$, while leaving behind the value V_{01} . The right hand side represents the gain for the worker of rejecting the offer: it will get cleaned out for no surplus in the event, with probability 1/2, that the firm makes the second offer, yet it will get, also with probability 1/2, the entire surplus, by being paid perpetually a wage p_H . In equilibrium the firm makes the offer that barely gets accepted, thus equating the left and right hand sides. The next two equations in (6) must be satisfied by wages w_L^1 and w_L^2 , respectively, for analogous reasons.

Nevertheless, when bargaining to determine w_H^1 , there is a twist: the firm knows that it can get away with paying a salary determined in the same way

as above, call it w . But when $w < p_L$, the firm knows that this match is only temporary, as L will keep searching and eventually will find a better paid job, inducing H to quit. The firm has another option: it may avoid turnover, by offering H a wage $w_H^1 = p_L$ instead; the firm would be getting a smaller flow profit, but knowing that it will keep earning it forever. We colloquially refer to the latter choice as "bribing", as the firm in a way is offering an extra payment to the couple to induce L to stay at home.⁶

One has to keep track of whether or not bribing happens in equilibrium. In particular, one has to consider the profit gain, $g(w)$, that the firm extracts from bribing a worker H that would settle for a wage w , or

$$\frac{p_H - p_L}{r} - \frac{p_H - w}{r + \lambda} = \frac{\lambda p_H + wr - (r + \lambda)p_L}{r(r + \lambda)} \equiv g(w)$$

It will also be useful to define $\Omega_k(w)$ to be the value of having the spouse $k \in \{H, L\}$, off the equilibrium path, earning an arbitrary wage $w < p_{\sim k}$, while the other spouse searches for a better job. It turns out that

$$\Omega_k(w) = \frac{h}{r} + \frac{4rw + 2\lambda p_{\sim k}}{r(2r + \lambda)}$$

When solving for w_H^1 one needs to analyze whether bribing would happen not only in the observed first offer by the firm, but also on the unobserved (off the equilibrium path) second stage of the bargaining game. In order to do this, it is useful to define some alternative values for w_H^1 , called \bar{w} , w^* and \tilde{w} , as follows: \bar{w} is the minimum wage that H is willing to accept in the second round of the bargaining game; w^* is the lowest wage that H would be willing to accept in the first round of the same game, given she is expecting that, if the firm had a chance to make the second offer, it would offer \bar{w} and extract all the surplus; \tilde{w} is the least wage H is willing to accept in the first round, given an expectation that the firm would make a bribing offer of p_L in the second round. Clearly $\tilde{w} > p_L$, so the first offer would never be a bribing

⁶Notice that this dilemma only arises when the firm is bargaining with H and L is still eligible to search, that is, over w_H^1 . If the situation was reversed, and the negotiation involved L instead of H , the firm would know that anything that is profitable to offer L will be topped eventually by H 's employer, so there is no way of avoiding turnover: the only sufficient drive deliver the firm negative profits. And in the event where firm and worker bargain while the spouse is employed, –that is, when determining w_H^2 or w_L^2 – the firm would know that bribing is not necessary, as the couple has no outside search option after the employed spouse quits and leaves the labor market.

offer if the firm would bribe in the second stage of the bargaining. These values are implicitly defined by

$$\begin{aligned}
\Omega_H(\bar{w}) &= V_{00} \\
\Omega_H(w^*) - V_{00} &= \frac{1}{2} \left[\frac{2p_H + h}{r} - V_{00} \right] \\
\Omega_H(\tilde{w}) - V_{00} &= \frac{1}{2} \left[\frac{2p_H + h}{r} - V_{00} \right] + \frac{1}{2} \left[\frac{2p_L + h}{r} - V_{00} \right]
\end{aligned} \tag{7}$$

Consider now w_H^1 . We know that when $\bar{w} < p_L$ and $g(\bar{w}) > 0$, agents will assume that the second offer by a firm would be a bribing one, and thus the first offer is $w_H^1 = \tilde{w} > p_L$. Alternatively, if $\bar{w} < p_L$ and $g(\bar{w}) \leq 0$, the firm would not bribe in the event of a second offer, so the first offer will need to be at least w^* . In this case, the firm's first offer would be a bribing one ($w_H^1 = p_L$) if $w^* < p_L$ and $g(w^*) > 0$, and it would not be a bribing one ($w_H^1 = w^*$) if $w^* < p_L$, $g(w^*) \leq 0$ or $p_L \leq w^*$. Finally, if $p_L \leq \bar{w}$, then the first offer must be $w_H^1 = w^*$. So

$$w_H^1 = \begin{cases} \tilde{w} & \text{if } \bar{w} < p_L \text{ and } g(\bar{w}) > 0 \\ w^* & \text{if } \bar{w} < p_L, g(\bar{w}) \leq 0 \text{ and } w^* < p_L, g(w^*) \leq 0 \text{ or } p_L \leq w^* \\ & \text{or if } \\ & p_L \leq \bar{w} \\ p_L & \text{if } \bar{w} < p_L, g(\bar{w}) \leq 0, w^* < p_L \text{ and } g(w^*) > 0 \end{cases} \tag{8}$$

Definition 9 An equilibrium is a combination value functions V_{00}, V_{10}, V_{01} , search strategies $\phi = (\phi_0, \phi_1, \phi_2)$ and wages $w = (w_H^1, w_L^1, w_H^2, w_L^2)$ that satisfy (4), (5), (6) and (8), given (7).

The pursuit for equilibria simplifies significantly thanks to the following:

Lemma 10 In all equilibria, $\phi_0 = \phi_1 = 1$

Proof. a) Obviously, $\phi_0 = \phi_1 = 0$ could never be a best strategy, because a deviation where either agent searches would increase expected income at no cost. b) The search strategies $\phi_0 = 1, \phi_1 = 0$ cannot be optimal because

they imply $\min\{V_{10}, V_{01}\} < V_{00}$. c) the same contradiction emerges when $\phi_0 = 0, \phi_1 = 1, \phi_2 = 1$. d) Finally, $\phi_0 = 0, \phi_1 = 1, \phi_2 = 0$ requires that $V_{01} \geq V_{00} \geq V_{10}$, $w_H^1 \leq p_L$ and $\frac{p_H - p_L}{r} \leq \frac{p_H - w_H^1}{r + \lambda}$, but it turns out that $V_{00} \geq V_{10}$ establishes a upper bound for p_L , while $w_H^1 \leq p_L$ establishes a lower bound for p_L , and the latter bound is higher than the former, so there are no solutions. ■

The Lemma simplifies matters quite significantly, paving the way to the following:

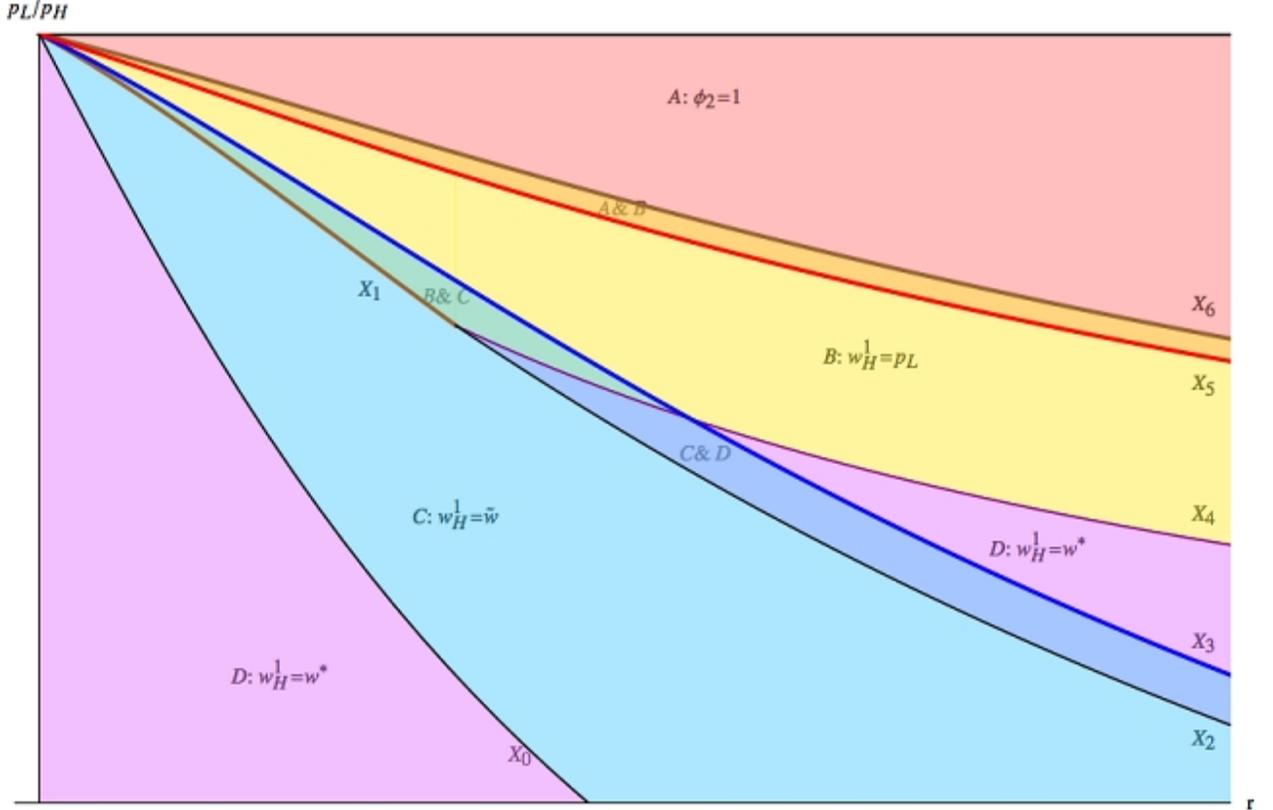
Proposition 11 *There is always an equilibrium. For couples that are very symmetric ($p_L \sim p_H$) the firm never bribes, and after the first spouse finds a job the second spouse always keeps searching ($\phi_2 = 1$). For intermediate values of p_L/p_H the equilibrium involves bribing, $w_H^1 = p_L$, so after H finds a job L drops off the labor market ($\phi_2 = 0$). For low values of p_L/p_H , the wage that H gets without bribing automatically takes L off the market, so ($\phi_2 = 0, w_H^1 > p_L$). These existence regions overlap, so for some couples there may be multiple equilibrium offers by the firm and search strategies by the couple. In particular, there may be both equilibria where the firm bribes and where it does not, and even two equilibria at different wages without bribing.*

Proof. Consider first the existence of an equilibrium where $\phi_2 = 1, w_H^1 = w^*$; solve (4), (6) and (7) to obtain candidate values for $V_{hl}, w_k^i, \bar{w}, \tilde{w}$ and w^* . The conditions (5) and (8) are $p_L > w^*$, $g(\bar{w}) \leq 0$, $V_{00} < \min\{V_{01}, V_{10}\}$ and $g(w^*) \leq 0$, which in turn translate into $p_L/p_H \geq \frac{2(r+\lambda)^2}{4r^2+5\lambda r+2\lambda^2}$, $p_L/p_H \geq \frac{3\lambda(r+\lambda)}{2r^2+4\lambda r+3\lambda^2}$ and $p_L/p_H \geq \frac{2(r+\lambda)(r+3\lambda)}{4r^2+9\lambda r+6\lambda^2} \equiv x_5$, with the only binding inequality being the last one.

Follow the same logic for an equilibrium with bribing, where $\phi_2 = 0, w_H^1 = p_L$, and verify that the conditions cannot be satisfied when $g(\bar{w}) > 0$, so this is an equilibrium only when $g(\bar{w}) \leq 0$, $w^* \geq p_L$, $V_{00} < \min\{V_{01}, V_{10}\}$ and $g(w^*) \geq 0$. These translate into $x_1 \leq p_L/p_H \leq x_6$ when $2r \leq \lambda$ and $x_4 \leq p_L/p_H \leq x_6$ when $2r > \lambda$, where $x_1 = \frac{\lambda(4r+7\lambda)}{4r^2+6r\lambda+7\lambda^2}$, $x_4 = \frac{2(r+\lambda)^2}{4r^2+5r\lambda+2\lambda^2}$ and $x_6 = \frac{2(r+2\lambda)^2}{4r^2+9\lambda r+8\lambda^2}$.

Notice that $x_6 > x_5$, so in the interval between them there is both an equilibrium with $\phi_2 = 0$ and $\phi_2 = 1$.

Identical procedures allow one to conclude that $\phi_2 = 0, w_H^1 = \tilde{w}$ in equilibrium only if $x_0 \leq p_L/p_H \leq x_3$, where $x_0 \equiv \frac{-4r^2-4\lambda r+4\lambda^2}{4r^2+\lambda^2}$ and $x_3 \equiv$

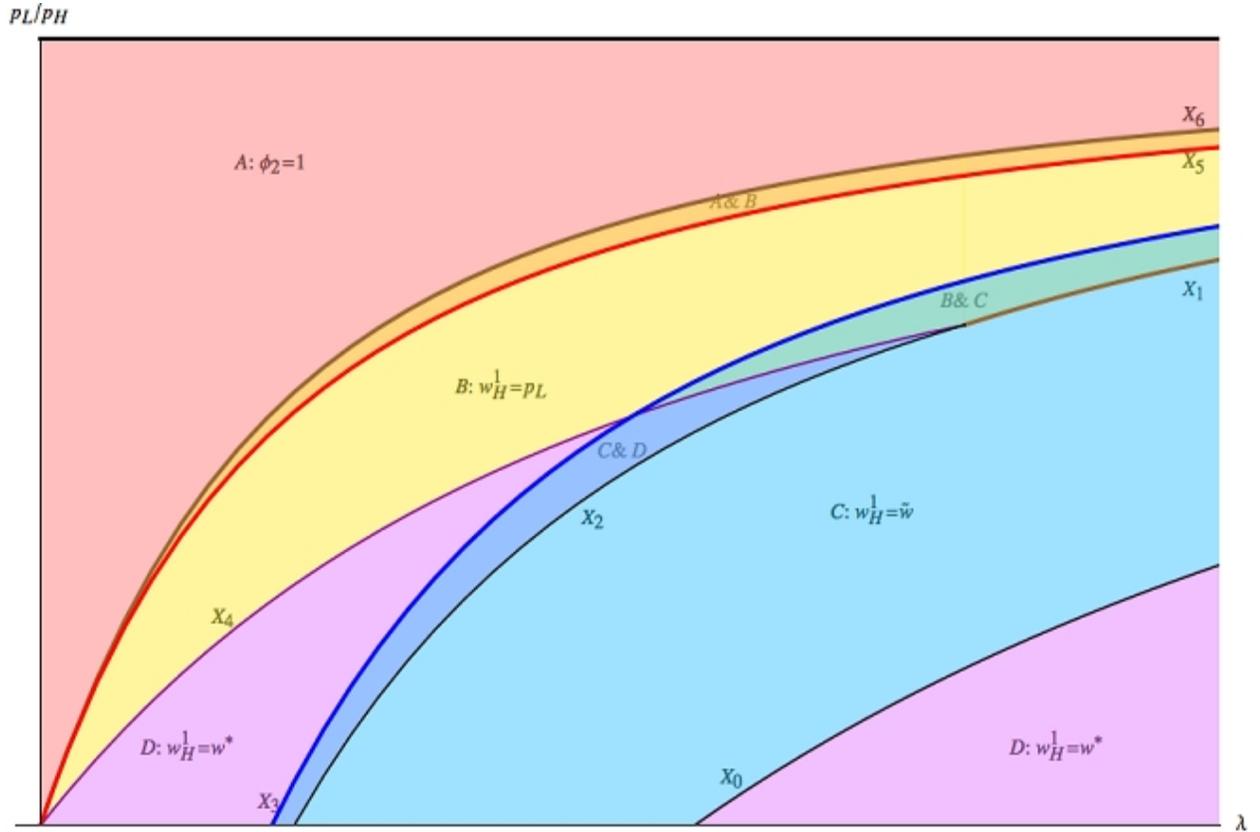


$\frac{\lambda(3r+4\lambda)}{2(r^2+2\lambda r+2\lambda^2)}$, and by the same token that $\phi_2 = 0$, $w_H^1 = w^*$ in equilibrium in two cases: one for $g(\bar{w}) < 0$ which exists only if $x_2 < p_L/p_H < x_4$, which only can happen if $2r > \lambda$ and where $x_2 \equiv \frac{3\lambda(r+\lambda)}{2r^2+4\lambda r+3\lambda^2}$; another for $\bar{w} > p_L$, which happens to exist whenever $p_L/p_H < x_0$.

Notice that x_1 and x_2 are both smaller than x_3 in the relevant intervals, so there are regions where multiple equilibria with $\phi_2 = 0$ exist. ■

The graphs illustrate the proposition.

Notice that for many couples (p_L, p_H) and parameter values there is a unique equilibrium, that in general involves $\phi_2 = 1$ (L searches when H is employed) when the couple is nearly symmetric (that is, p_L/p_H is close enough to 1), $\phi_2 = 0$ and bribing, for intermediate values of p_L/p_H , and $\phi_2 = 0$ without bribing when p_L/p_H is low. Furthermore, these areas overlap, so we have couples for whom both $\phi_2 = 0$ and $\phi_2 = 1$ correspond to equilibrium



search behavior, other couples for which there are equilibria with and without bribing, and others for which two different non-bribing wages may emerge.

The multiplicity of equilibria can be clearly observed in the figures. The orange region labelled $A\&B$ implies there are couples p_H, p_L where, if agents expect the firm to bribe, it is worthwhile for the firm to do so, and if they have the opposite expectation it is also self-fulfilling. In these cases, the worker is strictly better off in the equilibrium with the bribe. Similarly, in the green region labelled $B\&C$, if agents expect the firm to bribe it is rational for them to do so, and but if they expect to be paid an even higher wage that makes bribing unaffordable, the stronger bargaining position associated with that expectation makes it also an equilibrium. Finally, the blue region labelled $C\&D$ involves two non-bribing equilibrium levels for w_H^1 ; the difference between the two is that in one of them the agents expect that, off the

equilibrium path, if the first offer was rejected by a worker the second offer given by the firm would involve bribing, and if the other equilibrium they expect that it would not, with both expectations being rational.

4.1 Marriage

The analysis about the marriage market is similar to the one in the previous section: because V_{00} is strictly increasing on both p_H and p_L , and other features that differentiate among potential matches are binary, then all agents would rank between two arbitrarily selected spouses in the same way. This is a sufficient condition for an assortative equilibrium where the population into sets S_1, \dots, S_n according to their productivity, and agents in S_i only marry other agents in S_i . This would have the exact same implications as before: if finding a spouse is difficult, we will see some very heterogeneous couples, but as μ/r gets large, people only marry those compatible partners with very similar productivities to their own, and $p_L/p_H \rightarrow 1$.

Notice that in this last case we would only observe couples where bribing does not take place and couples always keep searching until the first one to find a job is the one who stays at home. This presents two possible sources of inefficiency. The first one is that in about half the families we will see the more productive spouse staying at home and the least productive working in the market. This represents a loss because both are equally good at home production, but not in the market; the loss, on the other hand, is not very big because the difference in productivities is small. The second one is probably more important: very productive individuals marry each other, only to have half of them staying at home, while very unproductive individuals also marry each other, to have half of them in the market. If the matching was not assortative, and instead couples married agents with very different productivities, one could have most the highly productive individuals working and viceversa, with a higher average value across society. The planner would not only allow, but probably compel, agents with $p_i \sim \bar{p}$ to marry partners very close to \bar{p} .

This inefficiency arises in part because we have chosen to make assumptions (namely, $\alpha = 1$) that emphasize the complementarity between home work and market work. If one allowed some complementarity between the market work of the two spouses instead (for instance, if flow payoff, as a function of p_H and p_L , was homogenous of a degree larger than one) then we would still observe assortative matching in equilibrium, but that feature

would be more compatible with optimality. By making p_L irrelevant when both agents work, and making one agent staying at home always optimal, we shut down this possibility. But the underlying message (that if one persons staying at home is a common enough optimal move across the whole space of the marriages observed in equilibrium, then welfare would increase if one could swap L in a very productive couple with H in a very unproductive one) will still carry some truth given some degree of complementarity between home and market work, as long as the very general conditions for an assortative equilibrium hold.

5 Conclusions

We have developed a model where the choice of marriage partner is endogenous, and once the couple is formed, it jointly decides its labor supply and home production. We find that the equilibrium involves different labor search strategies for different couples, and that often married agents –even the more productive spouse within the household, or somebody who has relatively high productivity among the population– stay at home. Couples with similar productivities to each other tend to choose strategies where both spouses do the same thing, while asymmetric couples tend to have asymmetric strategies. The latter kinds of couples tend, in equilibrium, to be less abundant (due to the assortative nature of equilibria), and more so as the technology for meeting potential spouses improves.

The findings about who marries whom tend to reconcile the results in Schwartz and Mare (2005), but the implications about income inequality do not necessarily follow, since in any equilibria where the two spouses in the couple behave symmetrically, in about half the households at any given time the less productive spouse is in the market and the more productive one stays at home. This means the income distribution among households may or may not be more unequal than the productivity distribution among individuals. Thus, the results in Cancian et.al (1993) are also consistent with our theoretical results.

We find that there may be multiple equilibria regarding wages, and also that unambiguously agents get better wage offers when their spouses are working, and the better paid they are, along the lines of Korenman and Newmark (1991). The multiplicity affects some couples, while for others their behavior and outcomes are deterministic. In some cases firms pay extra to

make sure that their employee's spouse stays at home, and in other cases this may or may not happen depending on expectations.

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