Marriage, Employment Participation and Home Production in Search Equilibrium

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Abstract

We model a marriage market where singles consider the prospects of employment and income of their potential spouses, and married couples make joint decisions on home production and labor participation. This double interaction between the marriage and labor markets is affected by search frictions in both. We characterize the job search-strategies of different couples; equal individuals have different behaviors depending on their spouses. When the search for mates is easy, people marry others with very similar productivity, and both spouses have the same behavior in the labor market. This natural outcome is socially inefficient as it takes some high productivity people off the labor market and viceversa. It also expands income distribution. Some empirical findings in the labor literature are supported theoretically here.

1 Introduction

In any labor market there is heterogeneity in workers wages and labor participation rates. Part of it is explained, of course, by differences in productivity: more productive individuals are likelier to pursue work, and also tend to receive a better wage. But, in general, other circumstances may also matter. In particular, the employment status, prospects and wages of one´s spouse may affect whether one seeks for work, as one shares income and efforts towards home production with that spouse. Married to a high earner, one is likelier to engage in home production, or to be more selective about which jobs to accept. Interestingly, since a person can get clear indications about a marital partner´s earning potential while pondering on the possibility of marriage, then not only their career is affected by the productive features of their spouse, but their choice of spouse is also affected by their potential careers.

In this paper, we develop a model where agents go first to a marital market and then a labor market. Agents choose their spouses taking into account their expected earnings, and once married the couple makes joint decisions regarding job search. Hence, the two-directional interaction described above is brought forward. Furthermore, spouses can collaborate not only by working and sharing their income, but also by specializing, one in market work, and the other in home production.

We find that in equilibrium, across the space of all possible couples, and each pair of spouses has a unique optimal strategy regarding labor search. There is a positive correlation in earning potentials among spouses, and when frictions in the marriage market are small, this correlation is very tight. Couples where both spouses have very similar productivity also have symmetric (within the couple) labor search strategies. Very heterogeneous couples behave asymmetrically.

In equilibrium, the population is divided in four classes: spouses with a similar (and high) productivity will constitute a high class where both will always stay in the job market, eventually sacrificing home production completely. If their productivity is similar but lower, they will choose to take turns to work, and at most generate one income. Other more heterogeneous couples will display strategies where the more productive member is always in the market, and the less productive one stays at home always, or almost always.

Other authors have looked previously at the interaction between the marriage and labor markets. In terms of theoretical work, we expand on Violante et al. (2012), who show how reservation wages are affected by marital status and joint search. Jaquemet and Robin (2013) study individual labour supply with a frictional marriage market. Bonilla and Kiraly (2013) study how the marriage wage premium arises as an equilibrium outcome in a model with frictional labor and marriage markets, while Bonilla et al. (2014) study the link between marriage and beauty wage premia in search equilibrium. We add to this literature as our main purpose is to study the consequences of the link between search for a partner and the facts that participation in the labor market is optional and consumption is, at least partially, a public good.

Regarding empirical work, our results reflect Schwartz (2010) who convincingly document that as the search technology has improved, the positive correlation in earning potential among spouses has increased, raising overall income inequality. This increased symmetry in the human capital that the spouses bring into the household reflects in an increasing similarity in their inputs and home production hours, as has been shown as far back as Cancian et. al (1993). Schwartz and Mare (2005) analyzes the data and reaches conclusions about the assortative nature of spouse choices, and about the implicit participation decisions, that interestingly fit our main theoretical results. We also obtain a theoretical explanation for Powell (1997) and Lovász and Szabó-Morvai, (2014) who find a positive effect of improved child care provision on female labor supply.

We describe the environment in Section 2, and derive the equilibrium in Section 3. We conclude in Section 4.

2 The environment

Time is continuous and continues forever. The population is a continuum of measure Ω_w of infinitely-lived women, and another of measure Ω_m of infinitely lived men. Both men and women discount future consumption at rate r. Each agent is characterized by an observable productivity $p \in [\underline{p}, \overline{p}]$, taken from the distribution function $F_m(p)$ in the case of men and $F_w(p)$ in the case of women.

When young, agents first enter a marriage market, where they can search (at a minimal but positive search cost ε) and encounter members of the opposite sex. For two people to be able to marry they require to be compatible (that is, all aspects of the relationship besides work and income, like attraction, personality, etc.), and not all potential couples are such. We assume that compatibility is a binary characteristic of the couple rather than the individual, uncorrelated with productivity, and not a matter of degree (in other words, if I like you then you like me, and while we could both also like other people out there, we would not like them more or less). These meetings between men and women emerge through a Poisson process. For a searching man, compatible women are encountered with an arrival rate $\mu_m = \mu\Omega_w$, and women find compatible men with arrival rate $\mu_w = \mu\Omega_m$.

Upon meeting a potential compatible candidate of the opposite sex, agents also observe their productivity, and they then decide whether or not to enter a permanent monogamous relationship, which emerges if doing so is strictly mutually agreeable; otherwise, they keep searching for another spouse. We also assume that agents can only entertain one suitor at a time, and need to give up a match in order to encounter other matches. When a couple marries, two clones of the newlyweds take their place in the marriage market.

Only once they have married do agents enter the labor market. When searching, jobs are found at another Poisson process, with arrival rate λ. Jobs pay as wages the worker´s marginal productivity, and are "indivisible" –or exclusively full-time– employment, in the sense that the number of hours worked is not variable.[1] With arrival rate $\delta > 0$, the job exogenously ends.

Spouses share income and home production; once married, preferences correspond to the couple, not the individual spouses. The value of home production increases with income with a marginal effect denoted α (that, for instance, enables to acquire household goods that complement home work), but requires that at least one member of the couple is not working. A second unemployed member of the family would be a waste, generating no income and adding nothing to the value of home production. Hence, instantaneous flow utility is

$$U = \begin{cases} p + P & & \text{both work and earn } p \text{ and } P \\ p + h + \alpha p & if & \text{one works for wage } p \text{ and the other does not work} \\ h & & \text{neither is employed} \end{cases}$$

Searching for production opportunities carries a cost ε; we assume $\varepsilon > 0$ but look at the limit case where $\varepsilon \longrightarrow 0$. This infinitesimal cost implies that agents will search only when they expect a strictly positive surplus from the market.[2].

[1] Here, as in much of the macro literature and also, for instance, in Rogerson and Wallenius (2012), individuals either work full time or not at all. Making the number of hours worked an endogenous variable (as in Rogerson and Wallenius (2013)) will probably not change the general meaning of the main results.

[2] Assuming costly search will keep us from having to analize mixed-strategy equilibria where agents who are indifferent between going to the market or not randomize the decision. Here, if you are indifferent about the outcome of search, you choose not to search, to avoid the cost.

Notice we do not need to make the same assumption of costly search for the marriage market. In fact, the proofs below are cleaner when we assume that men or women who are strictly indifferent between accepting or rejecting a particular partner always reject and keep searching, for an alternative the leaves them strictly better off.

3 Equilibrium

Due to the sequential nature of the problem, we can work out the labor market choices and performance of any possible couple (whether in equilibrium such a couple would exist or not). Then, given the payoffs obtainable in different matches, we look at the spouse-searching strategies of men and women.

For now, with no loss of generality, we will label H the spouse with a weakly higher productivity, and L the other spouse; whether H is the man or the woman will of course vary across couples, and is irrelevant for now. Their productivities will be denoted p_H and $p_L \leq p_H$. The value functions that correspond to their circumstances are denoted V_{HL}, where H (or L) take the value 1 when the spouse H (or L) has a job, and 0 when not. Needless to say, these functions V_{HL} are specific to each couple, as another pair with different productivities would enjoy different payoffs. Then,

$$\begin{aligned} rV_{00} &= h + \phi_0 \lambda \left(V_{10} - V_{00}\right) + \phi_1 \lambda \left(V_{01} - V_{00}\right) \\ rV_{10} &= (1+\alpha)p_H + h + \delta \left(V_{00} - V_{10}\right) + \phi_2 \lambda \left(V_{11} - V_{10}\right) \\ rV_{01} &= (1+\alpha)p_L + h + \delta \left(V_{00} - V_{01}\right) + \phi_3 \lambda \left(V_{11} - V_{01}\right) \\ rV_{11} &= p_H + p_L + \delta \left(V_{01} + V_{10} - 2V_{11}\right) \end{aligned} \tag{1}$$

The first equation tells us that the flow value of a couple where neither has a job is given by the value of home production (which, when nobody is getting an income, is just h), plus two factors related to their search behavior. First, if H is searching (with probability ϕ_0), the arrival rate λ of production opportunities that deliver the surplus $V_{10} - V_{00}$. Second, if L is searching (with probability ϕ_1), the arrival rate λ times the surplus $V_{01} - V_{00}$. The second equation tells us that a couple where only H works enjoys income p_H, plus the fruits of the home production of L (augmented by the income generated by H, or $h + \alpha p_H$), plus the arrival δ of the destruction of H´s job, times the implied net loss $(V_{00} - V_{10})$, plus, if L is searching for a job (with probability ϕ_2), the arrival λ of the surplus $V_{11} - V_{10}$. The other two equations can be understood analogously, given ϕ_3 is the probability H would search for a job when L is working.

For couples to behave optimally, it must be the case that at every chance they only search for a job if it improves their condition, so

$$\begin{aligned}
\phi_0 &= \begin{cases} 1 & \text{if} \quad V_{10} > V_{00} \\ 0 & \quad otherwise \end{cases} \\
\phi_1 &= \begin{cases} 1 & \text{if} \quad V_{01} > V_{00} \\ 0 & \quad otherwise \end{cases} \\
\phi_2 &= \begin{cases} 1 & \text{if} \quad V_{11} > V_{10} \\ 0 & \quad otherwise \end{cases} \\
\phi_3 &= \begin{cases} 1 & \text{if} \quad V_{11} > V_{01} \\ 0 & \quad otherwise \end{cases}
\end{aligned} \tag{2}$$

Definition 1 *For all couples (H, L) an optimal job-search strategy is a combination of values $V = (V_{00}, V_{01}, V_{10}, V_{11})$ and labor search probabilities $\phi = (\phi_0, \phi_1, \phi_2, \phi_3)$ that satisfies the Bellman equations (1) and incentive compatibility conditions (2).*

In principle, ϕ could take 16 different values, but the set of possible situations narrows quite a bit thanks to the following:

Lemma 2 *In any optimal strategy, a) $\phi_0 = 1$, and b) $\phi_2 = 1 \Longrightarrow \phi_1 = \phi_3 = 1$.*

Proof. Recall that there are no mixed strategies so the values $\phi_i \in \{0, 1\}$. a) Clearly, $\phi_0 = 0$ cannot be an optimal strategy when $\phi_1 = 0$, because the couple can raise their income with no sacrifice in home production if at least one of the members gets a job. On the other hand, $\phi_0 = 0$ while $\phi_1 = 1$ cannot be the optimal strategy since it is obviously dominated by $\phi_0 = 1, \phi_1 = 0$. Hence, there are no optimal strategies where $\phi_0 = 0$. b) When $\phi_2 = 1$, the lower-productivity spouse searches even though the higher-productivity spouse is employed. Then, the option of gaining p_L is worth giving up $h + \alpha p_H$. Clearly, if this is the case, it is also optimal for H to search when only L is working since $p_H > p_L$ (the prize is higher) and $h + \alpha p_L < h + \alpha p_H$ (the sacrifice is lower). Hence, $\phi_2 = 1 \Longrightarrow \phi_3 = 1$.

If the option of gaining p_L (the same prize) is worth giving up $h + \alpha p_H$ (because $\phi_2 = 1$), then it is also worth giving up just h. Hence, $\phi_2 = 1 \Longrightarrow \phi_1 = 1$. ■

The lemma indicates that, of the 16 possible combinations of ϕ_i that constitute alternative values for $\phi = (\phi_0, \phi_1, \phi_2, \phi_3)$, the 8 that include $\phi_0 = 0$

are not the best strategy for any couple, nor the 3 that include $\phi_2 = 1$ and either $\phi_1 = 0$ or $\phi_3 = 0$. Of the remaining 5 options, two ($\phi = (1,0,0,0)$ and $\phi = (1,0,0,1)$) can be collapsed into one -say $\phi = (1,0,0,\cdot)$, since they only differ in ϕ_3, which describes a choice that only happens if the L has a job, something that does not emerge on the equilibrium path if $\phi_1 = 0$ and $\phi_2 = 0$. Therefore, there are at most four possible types of optimal strategies, where $\phi = (\phi_0, \phi_1, \phi_2, \phi_3)$ assumes the values $(1,0,0,\cdot)$, $(1,1,0,0)$, $(1,1,0,1)$ or $(1,1,1,1)$.

The procedure to determine when each of these search strategies is the couple´s optimal behavior is very simple: assume one of the four candidate values of ϕ and substitute it in (1); then, solve for V_{HL}, and finally verify the parameter combinations for which (2) hold for that candidate ϕ, given those solutions for V_{HL}.

Lets analyze first the strategy $(1,1,1,1)$, where both spouses are always on the job market –either working or searching for work. This strategy leads to families that (in their preferred state) generate two incomes but no home production. Substituting $\phi = (1,1,1,1)$ in (1) and solving for V_{HL} yields:

$$\begin{aligned}
\Gamma V_{00} &= \lambda\left[r(1+\alpha) + 2\left(\delta + \alpha\delta + \lambda\right)\right](p_H + p_L) \\
&\quad + 2\delta(r^2 + 3r\left(\lambda + \delta\right) + 2\delta\left(\delta + 2\lambda\right))h \\
\Gamma V_{10} &= r\left[(1+\alpha)r + (2(\alpha+1)\delta + (2\alpha+3)\lambda)\right]p_H \\
&\quad + \lambda(2(\alpha\delta + \delta + \lambda) + r)(p_H + p_L) \\
&\quad + (2\delta + r)(\delta + 2\lambda + r)h \\
\Gamma V_{01} &= \left[(\alpha+1)r^2 + r(2(\alpha+1)\delta + (2\alpha+3)\lambda)\right]p_H \\
&\quad + \lambda(2(\alpha\delta + \delta + \lambda) + r)(p_H + p_L) \\
&\quad + (2\delta + r)(\delta + 2\lambda + r)h \\
\Gamma V_{11} &= \left[r^2 + r((2+\alpha)\delta + 3\lambda) + 2\lambda\left(\delta + \alpha\delta + \lambda\right)\right](p_H + p_L) \\
&\quad + 2\delta(r + \delta + 2\lambda)h \\
\text{where } \Gamma &= r(\delta + \lambda + r)(2(\delta + \lambda) + r)
\end{aligned}$$

Although these expressions are messy, they are also straightforward, and one can apply them to derive that

$$V_{01} > V_{00} \longleftrightarrow p_L > \frac{\lambda(h + \alpha p_H)}{2(\alpha+1)\delta + (\alpha+2)\lambda + (\alpha+1)r}$$

Similarly

$$V_{11} > V_{10} \longleftrightarrow p_L > \frac{(\delta + 2\lambda + r)(h + \alpha p_H)}{(\alpha + 2)\delta + 2\lambda + r} \equiv g_1(p_H)$$

and

$$V_{11} > V_{01} \longleftrightarrow p_L < \frac{((\alpha + 2)\delta + 2\lambda + r)p_H - (\delta + 2\lambda + r)h}{\alpha(\delta + 2\lambda + r)}$$

A little further exploration confirms that the first constraint is not binding, because it is laxer than the second constraint for all p_H. Furthermore, the second and third conditions coincide on the (p_H, p_L) plane on the 45° line, at the value

$$p_H = p_L = p^* = \frac{(\delta + 2\lambda + r)h}{(1 - \alpha)r + 2((1 - \alpha)\lambda + \delta)}.$$

For $p_H < p^*$, the two conditions cannot be satisfied jointly. For $p_H \geq p^*$, the third constraint is redundant with $p_L \leq p_H$. Hence, the strategy $\phi = (1, 1, 1, 1)$ is only the optimal job-strategies for couples where $p_H > p^*$ and $p_L > g_1(p_H)$. In other words, this is the behavior in couples where both spouses have similar, and high, productivity.

Analogous derivations can be obtained to characterize for which couples is each of our remaining options of ϕ the optimal job-search strategy. This analysis is presented in the Appendix, and its results are summarized in the following:

Proposition 3 *For all possible couples* $(p_H, p_L) \in [\underline{p}, \overline{p}] \times [\underline{p}, p_H]$, $\exists! \phi = (\phi_0, \phi_1, \phi_2, \phi_3)$ *that is optimal. In particular, there exist values* p^*, $p^o < p^*$ *and (linear, increasing) functions* $g_i(p_H)$, $i \in \{1, ..., 4\}$, *such that the optimal job-search strategy is*
a) $\phi = (1, 1, 1, 1)$ *if* $p_H > p^*$ *and* $p_L > g_1(p_H)$.
b) $\phi = (1, 1, 0, 0)$ *if* $p^o < p_H \leq p^*$ *and* $p_L > g_3(p_H)$ *or if* $p_H \leq p^o$ *and* $p_L > g_2(p_H)$.
c) $\phi = (1, 1, 0, 1)$ *if* $p_H > p^*$ *and* $p_L \in (g_4(p_H), g_1(p_H)]$, *or if* $p_H \in (p^o, p^*]$ *and* $p_L \in (g_4(p_H), g_3(p_H)]$
d) $\phi = (1, 0, 0, \cdot)$ *if* $p_H \leq p^o$ *and* $p_L \leq g_2(p_H)$, *or if* $p_H > p^o$ *and* $p_L \leq g_4(p_H)$.

Figure 1 illustrates these results:

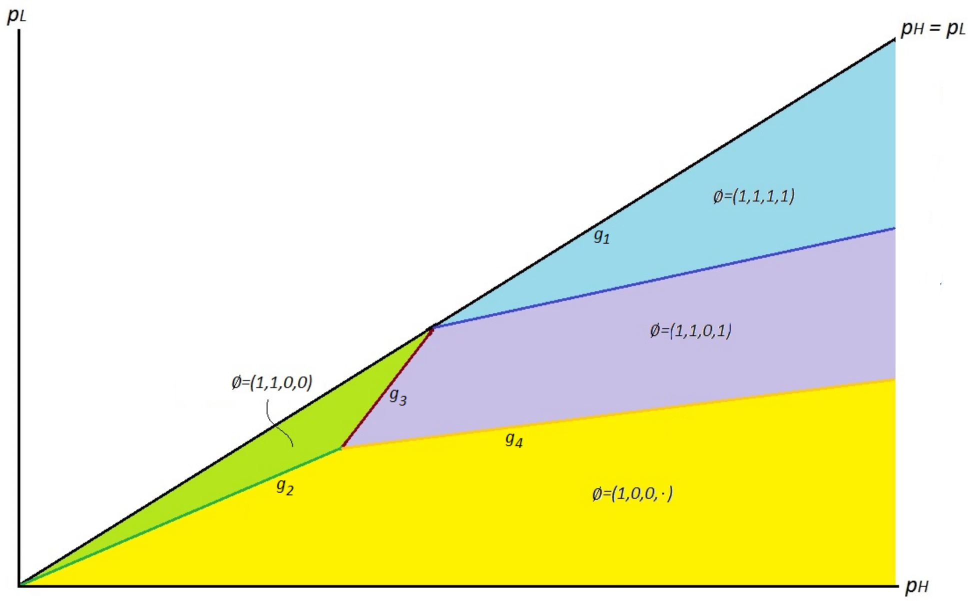

Figure 1: Regions in (p_H, p_L) space where different ϕ are the optimal strategy.

Lemma 4 *A couples utility is an increasing function of the productivity of each of its members. That is,* $V_{0,0}\left(p_H = \max\{p, \pi\}, p_L \min\{p, \pi\}\right)$ *is a weakly increasing, piecewise linear, weakly convex function of* p*, given* π *with slope* 0 *at* $p = 0$*.*

Proof. Let $\pi < p^{\circ}$. For $p \sim \underline{p}$, $\partial V_{0,0}\left(p_H = \max\{p, \pi\}, p_L = \min\{p, \pi\}\right)/\partial p = \partial V_{00}^{1000}/\partial p_L = 0$. For $p > g_2(\pi)$, that derivative becomes $\partial V_{00}^{1100}/\partial p_L > 0$, then for $p > \pi$ it is $\partial V_{00}^{1100}/\partial p_H = \partial V_{00}^{1100}/\partial p_L$, then for $p > g_2^{-1}(\pi)$ it is $\partial V_{00}^{1000}/\partial p_H > \partial V_{00}^{1100}/\partial p_H$. All these derivatives are non-negative and constant, and each is larger than the previous one.

To verify the same for $\pi \in [p^{\circ}, p^{*}]$, simply its a matter of verifying that $\partial V_{00}^{1101}/\partial p_H > \partial V_{00}^{1100}/\partial p_H = \partial V_{00}^{1100}/\partial p_L > \partial V_{00}^{1101}/\partial p_L > \partial V_{00}^{1000}/\partial p_L$, the appropriate values for $\partial V_{0,0}\left(p_H = \max\{p, \pi\}, p_L = \min\{p, \pi\}\right)/\partial p$ as p moves from $[\underline{p}, g_4(\pi)]$ to $[g_4(\pi), g_3(\pi)]$ to $[g_3(\pi), \pi]$ to $[\pi, g_3^{-1}(\pi)]$ to $[g_3^{-1}(\pi), \overline{p}]$.

Finally, to check the case where $\pi > p^{*}$, verify $\partial V_{00}^{1101}/\partial p_H > \partial V_{00}^{1111}/\partial p_H = \partial V_{00}^{1111}/\partial p_L > \partial V_{00}^{1101}/\partial p_L > \partial V_{00}^{1000}/\partial p_L$, which in turn corresponds to $\partial V_{0,0}\left(p_H = \max\{p, \pi\}, p_L = \min\{p, \pi\}\right)/\partial p$ as p moves from $[\underline{p}, g_4(\pi)]$ to $[g_4(\pi), g_1(\pi)]$ to $[g_1(\pi), \pi]$ to $[\pi, g_1^{-1}(\pi)]$ to $[g_1^{-1}(\pi), \overline{p}]$. ■

Notice we find that symmetric couples have symmetric strategies, and viceversa, in the sense that when the difference in productivity between husband and wife is small, the optimal job search behavior is the same for both.

Notice, for instance, the two regions adjacent to the 45° line, where $p_L \sim p_H$. At the top, a marriage of two similarly productivity people keeps them both in the labor market all the time. At the bottom, a marriage of similarly (low) productivity people keeps one of them –does not matter which– at home; when both are unemployed, both search, and when either one finds a job, the other one stops searching. Meanwhile, in the other regions, and especially in the region below where p_L is very low, the differences between the spouses are large, and their behavior is asymmetric. For instance, in the $\phi = (1, 0, 0, \cdot)$ region, one spouse is always in the market and the other is always at home.

Our results reflect the pattern identified in Powell and (1997) and Lovász and Szabó-Morvai (2014). Think for example of an increase in the value of home production (through an increase in unemployment benefits, or a decrease in childcare provision). In this case we observe a decrease in $g_3(p_H)$, while $g_4(p_H)$, $g_1(p_H)$, p^o and p^* increase. Further, the increase in $g_1(p_H)$ is higher than that of $g_4(p_H)$. This translates into the following qualitative results: The areas $\phi = (1, 1, 0, 0)$ and $\phi = (1, 0, 0, \cdot)$ both increase, reflecting that now more marriages will be such that one of the partners (the L in area $\phi = (1, 0, 0, \cdot)$) ends up not participating in the labour market. The area $\phi = (1, 1, 0, 0)$ also increases, but it does so at the expense of area $\phi = (1, 1, 1, 1)$ which shrinks. Once again, this means more partnerships in which the L stops participating in the labour market, and less marriages in which both partners remain in the labour market forever. A similar pattern obtains as one analyses an increase in α (brought about for example by a a decrease in the social provision of leisure opportunities).

The results also match the findings in Schwartz (2010). In our model, the individual behavior of different types of couples may augment the disparity across society. See for example, the contrast between a couple applying $\phi = (1, 1, 1, 1)$ and another choosing $\phi = (1, 1, 0, 0)$. Individually, each member in the first couple is more productive than each member in the second. Collectively, when both couples reach their desired state the former has twice the number of employed people than the latter, the differences in income become much larger. After discussing the equilibrium in the marriage market below, we address the links between family income distribution and efficiency in our equilibria.

What about the marriage market? Compatible, unemployed single people encounter each other at rate $\mu_k, k = m, w$. Denote $\widehat{V}_m(p)$ the value for single men with productivity p of searching in the marriage market. Obviously, for this man there is a reservation value, call it $R_m(p)$, such that he is not willing to marry a woman, even if she is compatible, with productivity lower than $R_m(p)$. For a woman with productivity p we can define $\widehat{V}_m(p)$ analogously.

Then

$$
\begin{aligned}
r\widehat{V}_m(p) &= \mu_m \int_{R_m(p)}^{R_w^{-1}(p)} V_{0,0}\left(p_H = \max\{p,\pi\}, p_L = \min\{p,\pi\}\right) dF_w(\pi) \quad (3)\\
r\widehat{V}_w(p) &= \mu_w \int_{R_w(p)}^{R_m^{-1}(p)} V_{0,0}\left(p_H = \max\{p,\pi\}, p_L = \min\{p,\pi\}\right) dF_m(\pi)
\end{aligned}
$$

The bounds in the integral simply imply that a single person of gender k with productivity p would not accept a marriage proposal from somebody with $\pi \leq R(p)$, nor get one from somebody with $\pi \geq R_{\sim k}^{-1}(p)$.

Definition 5 *An equilibrium in the marriage market is a pair of value function $\widehat{V}_m(p), \widehat{V}_w(p)$ and reservation strategies $R_m(p), R_w(p)$ such that (3) holds for all p.*

Of course, because all agents would rank any two (suitable) marriage candidates in the same order, we know from Burdett and Coles (2006) that, in any equilibrium for the marriage market the population will be assorted in classes, where the men in the top class marry women of the top class, men in the second class marry women in the second class, etc., with the possibility that, for some parameter values, some men or some women with very low productivity may never find someone who would take them[3].

Lemma 6 *(Burdett-Coles) There is a unique equilibrium of a marriage market, which takes the form of a partition of $[\underline{p}, \overline{p}]$ into sets S_i^m for the population of men, and S_i^w for the population of women,, where $S_1^k = (R_k(\overline{p}), \overline{p}]$,*

[3]It is an unfortunate feature of the model that these men or women that never marry also never work. This comes from our choie of sequence (first the marriage market, only then the labor market). We have explored the alternative where agents enter both markets simultaneously, but in this case the number of states to keep track of expands significantly, and the flavor of the results does not change. Hence, we opted for simplicity in this regard.

$S_2^k = (R_k(R_k(\overline{p})), R_k(\overline{p})]$, ...$S_i^k = (R_k \circ^i R_k(\overline{p}), R_k \circ^{i-1} R_k(\overline{p})]$, $k \in \{w, m\}$, where all agents of gender k with productivity $p \in S_i^k$ always marry the first compatible member of $S_i^{\sim k}$ that they encounter.
If the number of sets S_i^m for men is n, then the number of sets S_i^n for women will be $n-1$ (the least productive men never marry), n (everybody marries eventually) or $n+1$ (the least productive women never marry).
If μ/r is very low, $n = 1$. Also, n increases with μ/r and $n \to \infty$ as $\mu/r \to \infty$.

It is interesting to see that the link between inequality across agents and inequality across couples is behind an inefficiency in equilibrium that we derive and that reduces welfare. A social planner would try to generate a negative correlation between the productivity of spouses, to ensure that the less productive workers in society are as often as possible the less productive worker in their respective marriages, hence facilitating that they stay at home and specialize in home production, while the most productive workers in society are also the most productive workers in their couple, facilitating that they stay in the market. The equilibrium, by generating a positive correlation across spouses productivities, is keeping out of the labor market some highly productive agents (because they married even more productive spouses), and in the labor market some very unproductive agent (because they married even less productive spouses).[4]

In this regard, please note that as frictions disappear in the marriage market, then everybody marries their equal and all couples lie along the 45^0 line. In couples less productive than p^* this means that one relatively unproductive worker, but slightly more productive than their spouse, remains in the labor market; couples more productive than p^* are left –by choice– without the benefits of home production.

[4] A simple example is one where the population is dividided in two halfs, with productivities p_1 and p_2, where p_2/p_1 is a very high number. If μ and α are both high enough, in equilibrium the p_1 agents only marry each other, the p_2 agents only marry each other, and the labor force will be composed of half the population, of which again half would be p_1 and half would be p_2. In this case, the more productive half of society would enjoy utility $(1+\alpha)p_2$ and the other half would enjoy $(1+\alpha)p_1$. A social planner would prefer it if each p_1 married a p_2 (and viceversa), ensuring in that case that all the p_1 agents stay at home and all the p_2 agents work, which yields the higher utility $(1+\alpha)p_2$ for all agents. Hence, the same sorting mechanism that makes income distribution more skewed among couples than among individuals also leads to a loss of expected utility for all agents. Also, efficient sorting is likelier to emerge when productivity across agents is less variable.

Corollary 7 *The assortative nature of the marriage market equilibrium leads to an inefficient allocation in the labor market. In particular, some relatively productive individuals will stay at home if their spouse is even more productive, and some relatively unproductive individuals will stay in the labor market if their spouse is even less productive. An efficient outcome would require a negative correlation between the spouse's productivities, so that for every very productive man or woman there would be incentives to be always in the job market, married to a very unproductive spouse that stays always at home. Notice that one' s productivity at home is proportional to the productivity at work of one's spouse.*

Corollary 8 *If men and women are very similar (that is, both genders have similar population sizes and similar distributions F_k), then for large levels of μ/r, almost all agents have very similar productivity to their spouse, and thus most couples belong to the sets where, in equilibrium, $\phi = (1,1,1,1)$ or $\phi = (1,1,0,0)$. As we consider lower levels of μ/r, and the sets S_i are less numerous but larger, (or, alternatively, if we allow for disparities between the population of men and the population of women) there are sometimes bigger productivity differences across spouses, and an increasing fraction of the couples population share the home burdens asymmetrically: $\phi = (1;1;0;1)$ and $\phi = (1,0,0,\cdot)$.*

Corollary 9 *The standard deviation of household per-member incomes is larger than the standard deviation of individual productivities, both because highly productive individuals marry each other, and because those couples have a higher average participation rate than other couples.*

Corollary 10 *If there are asymmetries in the distribution of productivities of men and women, $F_w \neq F_m$ (say, because the here-unmodelled opportunities for education are not equal), in general the less productive gender will have a lower participation rate.*

Corollary 11 *If there are differences in population size between men and women, $\Omega_m \neq \Omega_w$, everything else being symmetric, the gender with the higher population will be less selective about marriage partners (have a lower $R_k(p)$), have a higher average labor-participation rate (since many of them will marry partners of the opposite sex that are less productive, since they are less selective), be slower to marry, and likelier to have a low-class of individuals that never marry.*

4 Conclusions

We have developed a model where the choice of marriage partner is endogenous, and once the couple is formed, it jointly decides its labor supply and home production. We find that the equilibrium involves different labor search strategies for different couples, and that often married agents -even the more productive spouse within the household, or somebody who has relatively high productivity among the population- stay at home. Couples of spouses with similar productivities to each other tend to choose strategies where both spouses do the same thing, while asymmetric couples tend to have asymmetric strategies. The latter kinds of couples tend, in equilibrium, to be less abundant (due to the assortative nature of equilibria), and more so as the technology for meeting potential spouses improves.

We find that the results we underscore in the Corollaries in Section 3 match a number of findings in the empirical literature. In addition to the facts mentioned in the Introduction, the findings about who marries whom tend to reconcile the results in Schwartz and Mare (2005), but the implications about income inequality do not necessarily follow, since in any equilibria where the two spouses in the couple behave symmetrically, in about half the households at any given time the less productive spouse is in the market and the more productive one stays at home. This means the income distribution among households may or may not be more unequal than the productivity distribution among individuals. Thus, the results in Cancian et.al (1993) are also consistent with our theoretical results.

5 Appendix

Here we prove Proposition 1.

We apply the same procedure that we used in the text for the strategy $\phi = (1, 1, 1, 1)$, now to the other three candidate strategies (not ruled out by Lemma 1): $\phi = (1, 1, 0, 0)$, $\phi = (1, 1, 0, 1)$, and $\phi = (1, 0, 0, \cdot)$.

Consider first $\phi = (1, 1, 0, 0)$. In this case, the value functions become

$$\begin{aligned}
V_{00} &= \frac{(1+\alpha)\lambda(p_L + p_H) + (r + 2\lambda + \delta)\, h}{r(\delta + 2\lambda + r)} \\
V_{01} &= \frac{\delta\lambda(1+\alpha)(p_L + p_H) + (1+\alpha)r(\delta + 2\lambda + r)p_L + (r+\delta)(\delta + 2\lambda + r)h}{r(\delta + r)(\delta + 2\lambda + r)} \\
V_{10} &= \frac{\delta\lambda(1+\alpha)(p_L + p_H) + (1+\alpha)r(\delta + 2\lambda + r)p_H + (r+\delta)(\delta + 2\lambda + r)h}{r(\delta + r)(\delta + 2\lambda + r)} \\
V_{11} &= \frac{\left[r^2 + 2(1+\alpha)\delta\lambda + r(2\lambda + (2+\alpha)\delta)\right](p_L + p_H) + 2\delta(\delta + 2\lambda + r)h}{r(2\delta + r)(\delta + 2\lambda + r)}
\end{aligned}$$

and the incentive compatibility conditions require only $V_{01} > V_{00}$ and $V_{10} \geq V_{11}$, since the latter makes $V_{01} \geq V_{11}$ redundant. This narrows down to

$$\begin{aligned}
p_L &> g_2(p_H) \equiv \frac{\lambda p_H}{\delta + \lambda + r} \\
p_L &\geq g_3(p_H) \equiv \frac{\left(\left[r^2 + r(\alpha\delta + 3\delta + 2\lambda) + \delta(\alpha + 2\delta + \alpha\lambda + 3\lambda)\right]\right) p_H - (\delta + r)(\delta + 2\lambda + r)h}{\delta\alpha(\delta + 3\lambda) + \delta\lambda + \alpha r^2 + 2\alpha r(\delta + \lambda)}
\end{aligned}$$

where we know that $g_3(p_H) < p_H$ only when $p_H < p^*$, as defined above, and that $g_2 \geq g_3$ if

$$p_H < p^\circ \equiv \frac{h(\delta + \lambda + r)}{\delta(\alpha + 2) + (1-\alpha)\lambda + r}$$

Therefore, the region where $\phi = (1, 1, 0, 0)$ is an optimal strategy is the one above g_2 for $p_H < p^\circ$, and above g_3 for $p_H \in [p^\circ, p^*]$.

Consider now the job search strategy is $\phi = (1, 0, 0, \cdot)$. Under this strategy,

$$\begin{aligned}
V_{00} &= \frac{\lambda(1+\alpha)p_H + (\delta + \lambda + r)\, h}{r(\delta + \lambda + r)} \\
V_{01} &= \frac{\lambda\delta(1+\alpha)p_H + r(1+\alpha)(\delta + \lambda + r)p_L + (r+\delta)\,(\delta + \lambda + r)h}{r(\delta + r)(\delta + \lambda + r)} \\
V_{10} &= \frac{(1+\alpha)(\lambda + r)p_H + (\delta + \lambda + r)h}{r(\delta + \lambda + r)} \\
V_{11} &= -\frac{r(\delta + \lambda + r)(r + 2\delta + \alpha\delta)(p_L + p_H) + 2(1+\alpha)\delta^2\lambda p_H + 2\delta(\delta + r)(\delta + \lambda + r)h}{r(\delta + \lambda + r)(\delta + r)(2\delta + r)}
\end{aligned}$$

and optimality requires $V_{01} \leq V_{00}$ and $V_{11} \leq V_{10}$. The former translates into $p_L \leq g_2(p_H)$; the latter translates into

$$p_L \leq g_4(p_H) \equiv \frac{\lambda(h + \alpha p_H)}{(r + 2\delta)(1 + \alpha) + \lambda}.$$

As it turns out, g_2 is the binding upper bound when $p_H \leq p^{\circ}$, and viceversa.

To conclude, consider now the job-search strategy $\phi = (1, 1, 0, 1)$. The value functions are straightforward to obtain yet rather messy, so we skip directly to the incentive compatibility conditions, which require simply $V_{10} \geq V_{11} > V_{01} > V_{00}$.

From the solutions of the value functions we derive that $V_{01} > V_{00}$ corresponds to $p_L > g_4(p_H)$. Meanwhile, $V_{10} \geq V_{11}$ holds if and only if $p_L \leq g_1(p_H)$, and $V_{11} > V_{01}$ if and only if $p_L < g_3(p_H)$. Since we know that the former is the binding constraint if $p_H > p^*$, and viceversa, we conclude that the couples for whom $\phi = (1, 1, 0, 1)$ is the best job-search strategy are those that satisfy

$$\begin{aligned} p_H &> p^* \text{ and } g_1(p_H) \geq p_L \geq g_4(p_H) \text{ or} \\ p^* &\geq p_H \geq p^o \text{ and } g_3(p_H) \geq p_L \geq g_4(p_H) \end{aligned}$$

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