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Offer Matching and Wage Dispersion

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Abstract

The paper considers a search-theoretic model of a labour market with on-the-job search, where firms post starting wages and choose whether to ignore or match outside offers. An offer matching equilibrium with wage dispersion is more likely when productivity is high. Multiple equilibria with different wage dispersion and outside offers policies are also possible.

Keywords: Labour market frictions; Wage policy; Wage dispersion
JEL classification: D82; J31; J63, J64

1 Introduction

An important explanation for the existence of equilibrium wage dispersion is based on search frictions and the heterogeneity of unemployed workers who differ in their value of leisure (see Albrecht and Axell, 1984). The idea is that a firm posting a wage faces a trade-off: higher wages lower profits per worker but attract more unemployed workers, whereas low wages increase profits but attract fewer job applicants. As shown in Gaumont, Schindler and Wright (2006), an interesting feature of this setup is that it can also generate multiple equilibria with wage dispersion.

We depart from the original model in two important ways. First, there is endogenous on-the-job search and firms cannot directly observe search effort. Second, employment contracts consist of a starting wage and a policy towards outside offers, where firms may choose to ignore or match these offers. This is similar to Kiraly (2007) and Postel-Vinay and Robin (2004), but those papers do not consider wage dispersion.

The present paper looks at equilibrium wage dispersion under the two policies towards outside offers. However, it is different from models such as Burdett and Mortensen (1998) where wage dispersion is an equilibrium outcome *because*

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of on-the-job search within an industry. Our results are driven by worker heterogeneity, albeit in the presence of outside offers.

The paper also provides a new insight into when multiple equilibria are possible and shows that in an equilibrium with offer matching the proportion of high-wage firms is higher than in an equilibrium with no offer matching. The intuition is as follows. Suppose low-wage firms match outside offers. Such a firm can manipulate the value it offers to the workers with low reservation values without affecting the participation constraint of the unemployed with high value of leisure. Other things equal, a lower starting wage coupled with the promise of future wage increases could be more profitable. For an equilibrium with non-degenerate wage dispersion to exist, the proportion of high-wage firms needs to be sufficiently high, so that the starting wage offered by low-wage firms is driven up accordingly.

2 The model

We follow quite closely the structure of the Gaumont, Schindler and Wright (2006) model, but we consider a segmented labour market. The primary sector has a continuum $[0, 1]$ of firms and a continuum $[0, L]$ of workers. All agents are risk neutral and have common discount rate r. The unemployed have different values of leisure (b) and we consider only the case with two types of workers $(b_1 < b_2)$, with L_i denoting the measure of type i unemployed. Primary firms produce output q $(> b_2)$ using a constant returns technology where labour is the only input. These firms hire only unemployed workers and contact occurs at rate . Firms post wages and maximize steady-state expected profits, so each firm is willing to hire as many workers as possible. Once employed, a worker chooses on-the-job search effort which is unobservable. If a worker chooses passive search (zero cost) he/she will contact a firm from the secondary market at rate λ_p. Alternatively, active search (at cost $c > 0$) leads to outside offers at rate λ_a $(> \lambda_p)$. At an exogenous rate δ, workers may leave the labour market altogether.[1]

Productivity in the secondary industry is also higher than b_2, but these firms hire workers directly from the primary sector. A raiding firm observes a worker's wage and employment status. The current employer is allowed to make counter offers should a poaching firm bid for the services of its worker. For simplicity, we assume that wages can never increase above w_2, possibly due to considerations of fairness within a firm.[2] Of course, a current firm may well decide not to match any outside offers and let a worker leave as soon as he/she is approached.

[1] In our segmented or dual markets setup, workers carry out job search in two labour sectors. The question of search in inter-linked frictional markets is itself an interesting one, with a growing literature. For example, Bonilla et al. (2017 and 2019) consider constrained search across a labour and a marriage market.

[2] A reasonable assumption if one interprets wage dispersion as a mixed strategy equilibrium where each firm offers both high and low wages. It does not affect our qualitative results.

2.1 The argument for policies towards outside offers

Throughout the paper we assume that firms in the primary market aim to discourage their workers from excessive on-the-job search. This is because active search leads to more frequent separations, and that is inefficient from the firm's point of view. With offer matching, a worker employed at a low wage has an incentive to attract outside offers. Once a rival firm is contacted, the bidding continues up to w_2, after which the worker is free to leave (and does so). As search effort is unobservable there is a moral hazard problem for the current employer, with workers actively searching for outside offers. This happens if the search cost is low enough:

$$c < (\lambda_a - \lambda_p)\left[\frac{w_2}{r+\delta} - W\right],$$

where W is the value of being employed at a firm that counters outside offers.

There are two ways of solving this incentive problem. First, the firm in the primary market may offer a high enough starting wage that deters workers from undertaking costly search. Alternatively, a firm may commit not to respond to any outside offers. Then, the worker leaves as soon as contact is made with a firm in the secondary market, but he/she gets paid only an ϵ more than the current wage as the raider doesn't need to bid more. This again reduces the incentive to search.

The rest of the paper focuses on the interesting case when either policy towards outside offers can form part of equilibrium contracts that implement passive on-the-job search. As shown in Kiraly (2007), a sufficient condition for this is

$$\lambda_p \leq \frac{(r+\delta)\lambda_a}{2(r+\delta)+\lambda_a}.$$

Given the above, we examine the existence of wage dispersion equilibria when all firms have the same (matching or not matching) policy towards outside offers.

2.2 Wage dispersion with no offer matching

Apart from a change in the overall separation rate ($\delta + \lambda_p$ here), this scenario is isomorphic to Albrecht and Axell (1984). With on-the-job search and no offer matching, one can construct a wage posting equilibrium with two wages: w_1^N and w_2. In such an equilibrium θ proportion of firms post the high wage w_2. Both types of firms earn equal expected profits

$$\Pi_j = \frac{\rho_j(q - w_j)}{r+\delta+\lambda_p}, \tag{1}$$

where ρ_j is the probability that a worker accepts the job that pays w_j. In particular, w_2 is just high enough to attract the unemployed with high leisure value. This also means that it is high enough to attract the unemployed with low value of leisure, so $\rho_2 = 1$. On the other hand, w_1^N is such that only workers with b_1 accept it and therefore $\rho_1 < 1$.

In equilibrium, the two wages are determined by the unemployment and employment values and the reservation property. They are $w_2 = b_2$ and

$$w_1^N = \frac{(r+\delta)b_1 + \quad \theta b_2}{r+\delta+ \quad \theta}. \tag{2}$$

Note that both wages are independent of λ_p: given the no-matching offers policy of the current employer, the wage never goes up even if the worker leaves the original employer. Secondly, w_1^N is a weighted average of b_1 and b_2, and it is higher than b_1 if and only if $\theta > 0$. If there is a chance of contacting a firm with a high wage, type 1 unemployed will not accept a wage just equal to their value of leisure.

These two wages are part of an equilibrium if $\Pi_2 - \Pi_1 = 0$, where $\Pi_2 - \Pi_1$ is proportional to the function

$$T(\theta) = (r+\delta+ \quad \theta)\left\{(q-b_2)\left[\delta L_1 + (\delta + \quad) L_2\right] - (q-b_1)\delta L_1\right\} - r \quad \theta L_1 (b_2 - b_1).$$

From $T(\theta) = 0$ we then get θ explicitly:

$$\theta = \frac{r+\delta}{\quad} \frac{\delta L_1 (q-b_1) - \left[\delta L_1 + (\delta + \quad) L_2\right]((q-b_2)}{\left[(r+\delta) L_1 + (\delta + \quad) L_2\right](q-b_2) - (r+\delta) L_1 (q-b_1)}. \tag{3}$$

Finally, $\theta \in (0,1)$ iff $q \in (\underline{q}^N, \overline{q}^N)$, where

$$\underline{q}^N = b_2 + \frac{\delta L_1 (b_2 - b_1)}{(\delta + \quad) L_2} \quad \text{and} \quad \overline{q}^N = \underline{q}^N + \frac{r \quad L_1 (b_2 - b_1)}{(r+\delta+ \quad)(\delta + \quad) L_2}. \tag{4}$$

2.3 Wage dispersion with offer matching

Consider again a potential equilibrium with wage dispersion, with a low w_1^M and a high w_2. This time, if an employed worker is under a wage w_1^M, the firm matches the initial offer and the bidding continues with offers and counter offers up to w_2, after which the worker leaves. Overall, the worker exits the primary industry at rate $\delta + \lambda_p$, so from the point of view of the firm the separation rate is not affected by the policy towards outside offers. Therefore, one can concentrate solely on the recruitment rate and the nature of the wage dispersion. Let $\sigma \in (0,1)$ be the fraction of firms posting the high wage w_2.

Denote by U_i the value function of an unemployed of type i. Then, standard considerations lead to the Bellman equations

$$\begin{aligned} rU_1 &= b_1 + \quad (1-\sigma)\left[W_1(w_1^M) - U_1\right] + \quad \sigma\left[W_1(w_2) - U_1\right] \quad \text{and} \\ rU_2 &= b_2 + \quad \sigma\left[W_2(w_2) - U_2\right], \end{aligned} \tag{5}$$

where $W_i(w)$ is the value of employment for a type i worker.

The two contracts are designed in such a way that $W_1(w_1^M) = U_1$ and $W_2(w_2) = U_2$. This makes use of the reservation wage property and the fact that W is increasing in wages. We have

$$rU_1 = b_1 + \quad \sigma \left[W_1(w_2) - U_1 \right] \qquad \text{and} \qquad rU_2 = b_2. \tag{6}$$

The value function for an employed captures the outcome of any competition for the services of that worker:

$$\begin{aligned} rW_1(w_1^M) &= w_1^M + \delta \left[U_1 - W_1(w_1^M) \right] + \lambda_p \left[V_1 - W_1(w_1^M) \right], \\ rW_2(w_2) &= w_2 + \delta \left[U_2 - W_2(w_2) \right] + \lambda_p \left[V_2 - W_2(w_2) \right], \\ rW_1(w_2) &= w_2 + \delta \left[U_1 - W_1(w_2) \right] + \lambda_p \left[V_1 - W_1(w_2) \right], \end{aligned}$$

where V is the value of employment in the secondary sector.

Given the above, we get

$$w_2 = b_2 \quad \text{and} \quad U_1 = \frac{r + \delta}{r(r + \delta + \quad \sigma)} b_1 + \frac{\sigma}{r(r + \delta + \quad \sigma)} b_2,$$

which in turn leads to

$$w_1^M = \frac{(r + \delta + \lambda_p) b_1 + (\quad \sigma - \lambda_p) \, b_2}{r + \delta + \quad \sigma}. \tag{7}$$

Note that w_1^M is once again a weighted average of b_1 and b_2. However, w_1^M depends on λ_p as well since a worker's wage is pushed up to w_2 as soon as contact is made with a raider. Crucially, w_1^M does not have to be greater than b_1. Even with a relatively low proportion of high-wage firms on the market, a type 1 unemployed may still accept an initial contract that pays a wage below his/her value of leisure. If an employed worker has a relatively good chance of contacting a raiding firm (that is, $\lambda_p > \quad \sigma$), the matching offers policy ensures that even a low starting wage is attractive enough. One might call this the "foot in the door" effect. A low wage firm that matches outside offers is now able to manipulate the value it offers to an unemployed by posting a lower starting wage w_1^M together with the implicit promise of a wage increase.

The optimal wage posting strategy is once again determined by the isoprofit condition. As before, $\rho_2 = 1$, but

$$\rho_1 = \frac{L_1(\quad \sigma + \delta)}{L_1(\quad \sigma + \delta) + L_2(\quad + \delta)}, \tag{8}$$

which makes use of the steady-state unemployment rates $u_1 = \frac{\delta}{\delta +}$ and $u_2 = \frac{\delta}{\delta + \quad \sigma}$.

After substituting ρ_1, ρ_2, w_1^M and w_2, one can show that $\Pi_2 - \Pi_1$ is proportional to $T(\sigma)$, where

$$T(\sigma) = (r + \delta + \quad \sigma) \left\{ (q - b_2) \left[\delta L_1 + (\delta + \quad) L_2 \right] - (q - b_1) \delta L_1 \right\} - \left[r \quad \sigma + \lambda_p (\delta + \quad \sigma) \right] L_1 (b_2 - b_1).$$

The condition $T(\sigma) = 0$ then gives

$$\sigma = \frac{r+\delta}{\ } \frac{\delta L_1(q-b_1) - [\delta L_1 + (\delta + \quad)L_2]\,(q-b_2) + \lambda_p \delta L_1(b_2-b_1)}{[(r+\delta)L_1 + (\delta + \quad)L_2)]\,(q-b_2) - (r+\delta)L_1(q-b_1) - \lambda_p L_1(b_2-b_1)}. \tag{9}$$

Proposition 1 *(a) There exists a non-degenerate wage dispersion equilibrium* σ *with offer matching iff* $q \in (\underline{q}^M, \overline{q}^M)$, *where*

$$\begin{aligned} \underline{q}^M &= b_2 + \frac{(r+\delta+\lambda_p)\delta L_1(b_2-b_1)}{(r+\delta)(\delta + \quad)L_2} \qquad \textit{and} \\ \overline{q}^M &= b_2 + \frac{\delta L_1(b_2-b_1)}{(\delta + \quad)L_2} + \frac{[r \quad + \lambda_p(\delta + \quad)]\,L_1(b_2-b_1)}{(r+\delta + \quad)(\delta + \quad)L_2}. \end{aligned}$$

(b) There also exists a multiple non-degenerate wage dispersion equilibrium θ *with no offer matching and* σ $(> \theta)$ *with offer matching iff* $q \in (\underline{q}^M, \overline{q}^M)$ *and*

$$\lambda_p < \frac{r \quad (\delta + \quad)}{\delta(r+\delta + \quad)}.$$

Proof. In any equilibrium with σ, the best responses are

$$\sigma = 0 \ \text{ if } \ T(0) < 0; \qquad \sigma = 1 \ \text{ if } \ T(1) > 0; \qquad \text{and } \ \sigma \in (0,1) \ \text{ if } \ T(\sigma) = 0$$

Assume without loss of generality that $b_1 = 0$. We need to check that $0 < \sigma < 1$. First, $\sigma > 0$ is true if the numerator and denominator in (9) are both positive or both negative. If they are positive, we have

$$\frac{\delta(1+\lambda)}{\delta +} > \frac{L_2(q-b_2)}{L_1 b_2} \quad \text{and} \quad \frac{L_2(q-b_2)}{L_1 b_2} > \frac{r+\delta+\lambda_p}{\delta +},$$

but that is impossible since

$$\frac{\delta(1+\lambda_p)}{\delta +} < \frac{r+\delta+\lambda_p}{\delta +}.$$

Hence, both the numerator and denominator are negative for

$$\frac{\delta(1+\lambda_p)}{\delta +} < \frac{L_2(q-b_2)}{L_1 b_2} < \frac{r+\delta+\lambda_p}{\delta +}.$$

The condition for $\sigma < 1$ is equivalent to

$$\frac{L_2(q-b_2)}{L_1 b_2} > \frac{(r+\delta)(1+\lambda_p)}{r+\delta +}.$$

Combining the latter two inequalities , and observing that

$$\frac{(r+\delta)}{r+\delta +} > \frac{\delta}{\delta +},$$

we get

$$\frac{(r+\delta)(1+\lambda_p)}{r+\delta+} < \frac{L_2(q-b_2)}{L_1 b_2} < \frac{r+\delta+\lambda_p}{\delta+},$$

which in turn defines $\underline{q}^M$ and $\overline{q}^M$.

Note that $\underline{q}^M > \underline{q}^N$ and $\overline{q}^M > \overline{q}^N$, so for a multiple equilibrium with both offer matching and no offer matching we need $\underline{q}^M < \overline{q}^N$, which leads to the condition on λ_p. It is straightforward to check that $\sigma > \theta$ for $q \in (\underline{q}^M, \overline{q}^M)$ and λ_p as defined above. ■

Finally, with $b_1 = 0$, the low wage in an offer matching equilibrium is

$$w_1^M = \frac{(r+\delta+\lambda_p)\left[(\delta + \quad)L_2(q-b_2) - L_1 b_2(\delta+\lambda_p)\right]}{(r+\delta)(r+\lambda_p)L_1}. \tag{10}$$

3 Discussion

In this paper we have extended the Gaumont, Schindler and Wright (2006) model. Our setup incorporates two salient features of a labour market with search frictions. First, workers differ in their value of leisure and search for better wages while employed. Second, firms post starting wages and also have policies that aim to discourage excessive on-the-job search.

When employed workers contact firms in the secondary market at a relatively high rate, there exists an equilibrium with wage dispersion and offer matching. An interesting feature of this equilibrium is that it only holds for relatively high productivities. When λ_p is high, offer matching is attractive from the point of view of the worker and hence a firm could post a very low starting wage. For a wage dispersion equilibrium to exist, this wage would have to be adjusted upwards a lot more, which is profitable only if the productivity is high enough. Overall, the model predicts that high productivity firms are more likely to have an offer matching policy.[3]

The paper also offers an insight into when separate wage dispersions coupled with different responses to outside offers could coexist as part of a multiple equilibria. If the arrival rate λ_p is low enough, one can get both offer matching and offer refusal as an equilibrium outcome, each policy paired with a separate wage dispersion. Comparing the two equilibrium outcomes, one finds that the proportion of high-wage firms is greater when firms match outside offers. As $\frac{\delta w_1^M}{\delta\sigma} > 0$, only when there are sufficiently many high-wage firms will the starting wage of a low-wage firm be pushed up enough in order to counterbalance the advantage created by the promise of matching future outside offers.

[3] This is very different from Postel-Vinay and Robin (2004). There, firms have heterogenous productivities and therefore a high productivity firm has an advantage in using an offer matching policy.

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