

Applications of Graph Width Parameters to Computational Complexity

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Many real-world computational problems can be modelled as problems on graphs. However, in most cases the resulting algorithmic graph problem is computationally intractable (for example, it is NP-hard or W[1]-hard) if any graph is allowed as input. In practical applications, we often find that the input for such problems is far from arbitrary, but has some additional properties or structure, and in some cases this structure is sufficient to construct efficient algorithms for these problems.

Thus it is useful to know what types of structure allow us to solve algorithmic problems efficiently. Ideally, we want to find properties that allow us to solve many different problems simultaneously, rather than solving each one in an ad hoc way. One of the techniques by which this can be done is by bounding width parameters of graphs. Some of these, such as treewidth and clique-width have been extensively studied, but others, such as mim-width and twin-width are more recent innovations. Width parameters allow us to measure how “well-structured” the graph under consideration is and, if the parameter is small, we can then use this structure to simultaneously construct efficient algorithms for wide classes of algorithmic graph problems, by the use of certain meta-theorems. Recent research has shown cases where even if such a parameter is not bounded on a class of graphs, it is often possible to combine bounding width parameters with additional tools to reduce problems to subclasses where the parameter is bounded, and thus achieve efficient runtimes (e.g. polynomial or fpt-time).

This project aims to systematically study the interplay between width parameters and other techniques to pinpoint exactly which types of structure allow us to solve computationally hard problems in an efficient manner (both from a classical complexity and a parameterized point of view). Finding such properties helps give insight into the mathematical structure of tractable instances. Conversely, if we find properties under which problems remain computationally hard, this increases our understanding about what it is that makes problems computational hard. An expected outcome of the project is to develop novel connections between Discrete Mathematics and Theoretical Computer Science by finding links between graph structure and algorithmic complexity.

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