

One sample Z and t Tests

These questions are for you to complete in your own time. Please use them for extra revision of the concepts discussed in lectures and practised in tutorials. For help with these questions, you can ask any of your tutorial leaders or visit Maths-Aid. Maths-Aid can be found in Room 1.16 on Level 1 of the Marjorie Robinson Library and can be contacted at mathsaid@ncl.ac.uk.

Normal Distribution

- 1) A random variable X follows a normal distribution with $\mu = 6$ and $\sigma = 1.24$. Calculate the following probabilities:
 - a) $P(X \leq 8)$
 - b) $P(X \leq 3)$
 - c) $P(X \geq 7)$
 - d) $P(7 \leq X \leq 10)$
- 2) A machine in a factory is responsible for filling boxes of cereal. The weight of cereal in each box is has a mean of 500g and a variance of 20g. A box is picked at random. Calculate the probability that it contains less than 490g of cereal.
- 3) When conducting a hypothesis test to check the means of samples, if the population standard deviation is known, we can use a When the population standard deviation is unknown, we use a
- 4) How can you tell if a hypothesis test should be one-tailed or two-tailed?

- 5) Resting heart rate is known to be 71 beats per minute on average, with a standard deviation of 4 beats per minute. A set of researchers believe that heart rate will increase in men when they are waiting to go in to a job interview. To test this hypothesis, a group of 9 men attending job interviews are fitted with a wireless heart rate monitor to wear on their chest in the hour preceding their interviews. Their average heart rates over this hour are shown in the table below.

Participant	Heart rate (bpm)
1	80
2	74
3	73
4	72
5	78
6	75
7	70
8	74
9	69

- a) Should a z-test or a t test be used to check if there is significant evidence to suggest heart rate increases in men while they are waiting to attend a job interview?
- b) Conduct the test at the 5% level and interpret your result.
- c) Calculate a 90% confidence interval for the population mean.
- 6) A visual research lab has purchased a digital colour blindness test from a company. Before they can use the test in their research, they must ensure it is properly calibrated. To do this they must check that they get the same results as the company when testing participants with no colour deficiencies. The company states that participants with healthy colour vision will score 15 on the test on average. The research lab tests 13 participants with healthy colour vision. On average they score 12 with a standard deviation of 3.6. Is their machine properly calibrated?

- 7) A psychology student, Sarah, is giving out sleep diaries to her university friends to monitor the number of hours of sleep they have each night. Sarah believes that university students sleep for 6 hours per night on average. Below is the data that they collected. The number of hours sleep per night for each student was averaged over a one month monitoring period. Is there any evidence to suggest that Sarah's belief is incorrect?

Participant	Hours of sleep per night
1	7.2
2	8.7
3	5.4
4	6.1
5	5.6
6	6.7
7	5.9
8	6.3
9	7
10	4.2

- 8) Previous research has found an auditory detection threshold of 3.24 dB for a certain auditory stimulus, with a standard deviation of 1.22 dB. This research used the method of constant stimuli. Using a staircase method with 16 participants, the auditory detection threshold for the same stimulus is found to be 2.89 dB. Is there any evidence using the new method to suggest the original result was incorrect?
- 9) A researcher reads a paper introducing a new test of executive function. The authors of the paper had tested 30 healthy volunteers and found that on average they score 51 on the new test. The researcher believes that patients with schizophrenia will score less than the normal average on this test of executive function. They test 22 patients with schizophrenia. The average score for the patients is 39 with a standard deviation of 4.3. Is there significant evidence at the 5 % level to support the researcher's claim?

Solutions

- 1) a) 0.9463
b) 0.0078
c) 0.2090
d) 0.2084

2) $P(X \leq 490) = P\left(Z \leq \frac{490-500}{\sqrt{20}}\right) = P(Z \leq -2.236) = 0.0127$. In other words, we would expect roughly 12.7% of boxes to weigh less than 490g.

3) When conducting a hypothesis test to check the means of samples, if the population standard deviation is known, we can use a z-test. When the population standard deviation is unknown, we use a t-test.

4) It will be 1-tailed if we are expecting the sample mean to be either significantly higher or significantly lower than the population mean. It will be 2-tailed if we are expecting the sample mean to be significantly different to the population mean (but we do not know whether we expect it to be higher or lower).

5) a) A z-test as we have the population standard deviation.

b) $H_0: \mu = 71$, $H_1: \mu > 71$, $\sigma = 4$, $\bar{x} = 73.8$

$$P(X \geq 73.8) = P\left(Z \geq \frac{73.8 - 71}{\frac{4}{\sqrt{9}}}\right) = P(Z \geq 2.167) = 1 - 0.9849 = 0.0151 < 0.05$$

There is significant evidence to reject the null hypothesis and hence there is significant evidence to suggest that waiting to attend an interview increases heart rate in men.

c)

$$\left(73.8 + 1.64 \times \frac{4}{\sqrt{9}}, 73.8 - 1.64 \times \frac{4}{\sqrt{9}}\right) = (71.70, 76.08)$$

6) We need to use a t-test as we don't have the population standard deviation. It will be 2-tailed as we are checking to see if the machine gives different values, rather than specifically lower or higher values. $H_0: \mu = 15$, $H_1: \mu \neq 15$, $s = 3.6$, $\bar{x} = 12$.

$$\frac{12 - 15}{\frac{3.6}{\sqrt{13}}} = -3.0046$$

The critical value for a 2-tailed t-test at the 95% level with 12 degrees of freedom is $2.179 < 3.0046$, therefore our result is significant and we can conclude that the machine is not calibrated properly.

7) We need to use a t-test as we don't have the population standard deviation. It will be 2-tailed as we are checking to see if the average length of sleep differs from 6, rather than if the average amount of sleep is specifically lower or higher than 6. $H_0: \mu = 6$, $H_1: \mu \neq 6$.

Firstly, the mean \bar{x} and the standard deviation s are calculated to be:

$$\bar{x} = 6.31, \quad s = \sqrt{\frac{13.129}{10 - 1}} = 1.208$$

This gives us a t-value of

$$\frac{6.31 - 6}{\frac{1.208}{\sqrt{10}}} = 0.8115$$

The critical value for a 2-tailed t-test at the 95% level with 9 degrees of freedom is $2.262 < 0.8115$, therefore our result is not significant and we have no evidence to reject the null hypothesis that students sleep for 6 hours on average.

8) 2-tailed z-test. $H_0: \mu = 3.24, H_1: \mu \neq 3.24, \sigma = 1.22, \bar{x} = 2.89$

$$P(X \leq 2.89) = P\left(Z \leq \frac{2.89 - 3.24}{\frac{1.22}{\sqrt{15}}}\right) = P(Z \leq -1.111) = 0.1333 > 0.025$$

Therefore, the result is insignificant and we retain the null hypothesis that $\mu = 3.24$.

9) 1-tailed t-test. $H_0: \mu = 51, H_1: \mu < 51, s = 4.3, \bar{x} = 39$

$$t = \frac{39 - 51}{\frac{4.3}{\sqrt{22}}} = 13.090$$

The critical value for a 1-tailed t-test at the 95% level with 21 degrees of freedom is $1.721 < 13.090$, therefore our result is significant and we conclude that there is evidence to suggest that people with schizophrenia score less than the general population.