Binomial Hypothesis Testing

These questions are for you to complete in your own time. Please use them for extra revision of the concepts discussed in lectures and practised in tutorials. For help with these questions, you can ask any of your tutorial leaders or visit Maths-Aid. Maths-Aid can be found in Room 1.16 on Level 1 of the Marjorie Robinson Library and can be contacted at mathsaid@ncl.ac.uk.

1) In the following questions, identify the parameters of the underlying binomial distribution. These are the number of trails (n) and the probability of success (p).

   a) A fair coin is flipped 20 times and the number of heads and tails is recorded.

   b) A card is pick at random from a deck 10 times with replacement. If the card is an Ace a 'hit' is recorded, if not a 'miss' is recorded.

2) Calculate the following probabilities for a binomial distribution with $n = 13$ and $p = 0.5$.

   a) $P(X = 5)$

   b) $P(X < 3)$

   c) $P(X \geq 12)$

   d) $P(6 < X < 9)$

3) A researcher is reading a paper on colour preference of 1-3 year olds. The paper states that when presented with identical blue and pink cubes, boys are more likely to select the blue cube than the pink cube. In fact, the paper claims that if a 1-3 year old boy is chosen at random and asked to pick a cube, the probability of them choosing the blue cube is $p = 0.8$. The researcher reading the paper has doubts about this claim, as they believe that boys of this age are equally likely to pick either cube. To test the claim, they present 10 randomly sampled 1-3 year old boys with a blue and pink cube and record the choices. Out of the 10 boys sampled, 3 of them chose the blue cube. Use a binomial hypothesis test at the 5% significance level to see if the researcher has found evidence against the claims of the paper.

4) A comparative behaviour psychologist is interested in whether male dogs form closer attachments to owners of a specific sex. To investigate this, the psychologist finds 12 heterosexual couples (living together and without children) who have recently purchased a male dog. After each couple has had the dog for a year the psychologist conducts a series of tests to determine which owner the dog has formed the greatest bond with. Out of the 12 dogs, they find 8 of them are closer to their male owner. Perform a binomial hypothesis test at the 5% level to determine whether this is a significant effect.

5) A new intervention technique has been developed to help smokers reduce the number of cigarettes they smoke per day. To test the effectiveness of this intervention, a sample of 13 smokers record the number of cigarettes smoked per day over a one month period. At the end of this period, all participants receive the intervention, after which the participants must again record the number of cigarettes smoked per day for the following month. The results are shown in the table below. Is there significant evidence that the intervention works? Use a sign test with $\alpha = 0.05$.

Participant	**Average number of cigarettes per day before intervention**	**Average number of cigarettes per day after intervention**
1	35	33
2	28	27
3	41	31
4	27	18
5	33	22
6	30	28
7	23	25
8	21	22
9	19	19
10	37	30
11	34	31
12	23	20
13	18	11

6) A pharmaceutical company is developing a new drug for the treatment of depression and they are keen to see if it has any effect on the speed of information processing. They have chosen to test this using the digit symbol substitution task (DSST), in which the numbers 1-9 correspond to different symbols and participants are required to enter the correct symbol in a box marked with a certain number. Below are the completion times of the DSST for 12 patients suffering from depression before and after treatment with the new drug.

   a. Which test would be the most suitable to check if the drug has a significant effect on information processing?

   b. Test, at the 5% level, whether the drug has a significant effect on information processing.

Patient	Completion time before drug (s)	Completion time after drug (s)
1	97	103
2	120	122
3	135	149
4	108	117
5	118	124
6	141	134
7	94	109
8	89	119
9	102	123
10	106	114
11	128	138
12	134	129

7) Caffeine is generally accepted to increase levels of alertness. As part of their dissertation, a student is looking at the effects of caffeine tablets on alertness by recording reaction times on an auditory task in 16 participants before and after taking a caffeine tablet. In the auditory reaction time task, participants are required to press a buzzer as quickly as they can upon hearing a tone. The median reaction time before caffeine was administered was 344 ms. The reaction times after caffeine are given in the table below. Using a median test, is there evidence to suggest that reaction time is decreased following administration of caffeine?

Participant	Reaction time after caffeine (ms)
1	300
2	316
3	298
4	345
5	380
6	453
7	342
8	468
9	452
10	367
11	310
12	349
13	298
14	276
15	401
16	303

Solutions

1) a) $n = 20, p = 0.5$
   b) $n = 10, p = \frac{1}{13} = 0.077$

2) a) ${}^{13}C_5 \times 0.5^5 \times 0.5^8 = 0.1571$
   b) ${}^{13}C_0 \times 0.5^0 \times 0.5^{13} + {}^{13}C_1 \times 0.5^1 \times 0.5^{12} + {}^{13}C_2 \times 0.5^2 \times 0.5^{11} = 0.0112$
   c) ${}^{13}C_{12} \times 0.5^{12} \times 0.5^1 + {}^{13}C_{13} \times 0.5^{13} \times 0.5^0 = 0.0017$
   d) ${}^{13}C_7 \times 0.5^7 \times 0.5^6 + {}^{13}C_8 \times 0.5^8 \times 0.5^5 = 0.3666$

3) $H_0: p = 0.8, H_1: p < 0.8$
$P(X \leq 3) = {}^{10}C_0 \times 0.8^0 \times 0.2^{10} + {}^{10}C_1 \times 0.8^1 \times 0.2^9 + {}^{10}C_2 \times 0.8^2 \times 0.2^8 + {}^{10}C_3 \times 0.8^3 \times 0.2^7 = 0.0009 \leq 0.05$ so there is significant evidence to reject the null hypothesis: there is significant evidence to suggest that the probability of boys choosing the blue cube is less than 0.8.

4) $P(X \geq 8 \text{ or } X \leq 4) = 0.3877 > 0.05$, so the effect is not significant.

5) $P(X \geq 10) = 0.0193 < 0.05$, so there is significant evidence to suggest the intervention reduces the number of cigarettes smoked per day.

6) A sign test. $P(X \geq 10 \text{ or } X \leq 2) = 0.0386 < 0.05$, so there is significant evidence to suggest the drug affects the speed of information processing.

7) $P(X \geq 8) = 0.5982$, so there is no evidence to suggest caffeine improves reaction times.